The problems on this list are not intended to be representative of all the kinds of problems which will appear on the final exam. Rather, they are “extra” sample problems.

At least 80% of the points on the real exam will be modifications of problems from the problems below, homework problems (particularly written homework), worksheet problems, sample exam problems, and problems from the real and sample midterms and their associated worksheets. Note, though, that the exact form of the functions to be expanded in series, limits and sums to be found, etc., could vary substantially, and the methods required to do them might occur in different combinations.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation in the book, in handouts, in files posted on the course website, and on the blackboard; use it. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Problems.

1. (12 points.) For the differential equation $y''(x) + y'(x) + 3xy(x) = 6\sin(x)$, find the terms through degree 4 of the power series solution centered at 0 and satisfying $y(0) = 1$ and $y'(0) = 7$.

2. (10 points.) Consider the function $f(x) = (e^x + 13)^{-1}$ for real numbers $x$. What is the radius of convergence of its Taylor series centered at $x = 10$? Explain your reasoning.

3. (10 points.) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{3^n x^{2n}}{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n - 1)}.$$

For any convergence test that you use, be sure to say why it applies.

4. (7 points.) Find the Taylor polynomial for the function $f(x) = e^{x^2} \sin(x)$ of degree 4 centered at $x = 0$. (Simplify but don’t multiply out the coefficients.)

5. (10 points.) Consider the power series $\sum_{n=0}^{\infty} \frac{(x - 5)^n}{6^{n+2}(n + 3)^4}$. You are told that its radius of convergence is 6. Given this, find its interval of convergence. For any convergence test that you use, be sure to say why it applies.
6. (6 points.) Define a sequence \((a_n)_{n=1}^\infty\) by \((a_n)_{n=1}^\infty = \left(\frac{5}{2}, \frac{6}{3}, \frac{7}{4}, \frac{8}{5}, \frac{9}{6}, \frac{10}{7}, \ldots\right)\). Find its limit (possibly \(\infty\) or \(-\infty\)), giving reasons, or explain why the sequence neither converges nor diverges to \(\infty\) or \(-\infty\).

7. (6 points.) Define \(f(x) = \cos(3x^4)\) for all real \(x\). Find \(f^{(12)}(0)\). (Remember to show your work. Simplify your answer but don’t multiply out powers, factorials, etc.)

8. (6 points.) Define a sequence \((a_n)_{n=1}^\infty\) by \(a_n = \frac{\sin(2n)}{n+1}\) for \(n = 1, 2, 3, 4, \ldots\). Find its limit (possibly \(\infty\) or \(-\infty\)), giving reasons, or explain why the sequence neither converges nor diverges to \(\infty\) or \(-\infty\).

9. (10 points.) Consider the function \(f(x) = (x^3 + 27)^{-1}\) for real numbers \(x\). What is the radius of convergence of its Taylor series centered at \(x = 2\)? Explain your reasoning.