Names and student IDs: ____________________________________________

1. Find the exact value of \( \lim_{n \to \infty} \frac{(n+1)!}{3n \cdot n!} \).

2. Set \( a_n = 1 - \frac{1}{n} \) for positive integers \( n \). Which of these statements is correct?
   (1) \( (a_n)_{n=1}^{\infty} \) is bounded.
   (2) \( (a_n)_{n=1}^{\infty} \) is strictly increasing. (The book has “increasing”.)
   (3) \( (a_n)_{n=1}^{\infty} \) is strictly decreasing. (The book has “decreasing”.)
   (4) \( (a_n)_{n=1}^{\infty} \) is strictly monotonic. (The book has “monotonic”.)

3. Set \( b_n = n + (-1)^n \cdot n + 2 \) for positive integers \( n \). Which of these statements is correct? (First, explicitly calculate the first few terms.)
   (1) \( (b_n)_{n=1}^{\infty} \) is bounded.
   (2) \( (b_n)_{n=1}^{\infty} \) is strictly increasing.
   (3) \( (b_n)_{n=1}^{\infty} \) is strictly decreasing.
   (4) \( (b_n)_{n=1}^{\infty} \) is strictly monotonic.

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4. Define a sequence \((a_n)_{n=1}^{\infty}\) by \(a_n = \sqrt{n}\) for \(n = 1, 2, \ldots\). You want to prove that \(\lim_{n \to \infty} a_n = \infty\). So, for example, for \(M = 137\) you should be able to find some integer \(N > 0\) such that for all \(n > N\) we have \(a_n > M\). Find such a value of \(N\), and show that it works. (You need *not* find the best value of \(N\).)

5. Define a sequence \((x_n)_{n=1}^{\infty}\) by
\[
x_n = \frac{2n + 1}{n}
\]
for \(n = 1, 2, \ldots\). You want to prove that \(\lim_{n \to \infty} x_n = 2\). So, for example, for \(\varepsilon = 0.2\) you should be able to find some integer \(N > 0\) such that for all \(n > N\) we have \(|x_n - 2| < \varepsilon\). Find such a value of \(N\), and show that it works. (You need *not* find the best value of \(N\).)

6. Define a sequence \((c_n)_{n=1}^{\infty}\) by
\[
c_n = \frac{n^2 - 10}{n^2}
\]
for \(n = 1, 2, \ldots\). You want to prove that \(\lim_{n \to \infty} c_n = 1\). So, for example, for \(\varepsilon = 0.1\) you should be able to find some integer \(N > 0\) such that for all \(n > N\) we have \(|c_n - 1| < \varepsilon\). Find such a value of \(N\), and show that it works. (You need *not* find the best value of \(N\).)