Recall: The series $\sum_{n=1}^{\infty} a_n$ is said to be absolutely convergent if $\sum_{n=1}^{\infty} |a_n|$ is convergent. (This is an even worse abuse of language than those we have seen so far: $\sum_{n=1}^{\infty} a_n$ is a number, and this number doesn’t depend on which particular series we summed to get it. However, the terminology is too well established to fight.)

Recall that an absolutely convergent series is convergent, but that a convergent series need not be absolutely convergent.

Note: One never says “absolutely divergent”.

1. For each of the following series, determine whether it is absolutely convergent and whether it is convergent. (For one of them, you won’t be able to tell. Which one is it?)

   a. $\sum_{n=0}^{\infty} \frac{3 + 7 \sin(n)}{7^n}$

   b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   c. $\sum_{n=1}^{\infty} \frac{\cos(e^n)}{n^2}$

   d. $\sum_{n=1}^{\infty} \frac{\cos(\sqrt{n})}{\sqrt{n}}$

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Recall the Ratio Test for convergence of the series \( \sum_{n=1}^{\infty} a_n \), in which all the summands are nonzero:

1. If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) exists and is less than 1, then \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent (and therefore also convergent).

2. If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) exists and is greater than 1, or if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty \), then \( \sum_{n=1}^{\infty} a_n \) is divergent.

3. In all other cases, that is, if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \) or if this limit fails to exist in any other way than diverging to \( \infty \), the Ratio Test says nothing. Try some other test.

2. For each of the following series, what does the Ratio Test say about its convergence? If the Ratio Test doesn’t apply, can you use other methods?

   a. \( \sum_{n=1}^{\infty} \frac{(-1)^n 15^n}{n!} \)

   b. \( \sum_{n=1}^{\infty} (-1)^{n-1} n \)

   c. \( \sum_{n=1}^{\infty} \frac{n^{16}}{6^n} \)

   d. \( \sum_{n=1}^{\infty} \frac{n}{n^3 + 1} \)