WORKSHEET: RATIO TEST

Recall the Ratio Test for convergence of the series \( \sum_{n=1}^{\infty} a_n \), in which all the summands are nonzero:

1. If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) exists and is less than 1, then \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent (and therefore also convergent).
2. If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) exists and is greater than 1, or if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty \), then \( \sum_{n=1}^{\infty} a_n \) is divergent.
3. In all other cases, that is, if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \) or if this limit fails to exist in any other way than diverging to \( \infty \), the Ratio Test says nothing. Try some other test.

1. For each of the following series, what does the Ratio Test say about its convergence? If the Ratio Test doesn’t apply, can you use other methods?

\[ \sum_{n=1}^{\infty} \frac{(-1)^n 15^n}{n!} \]
b. $\sum_{n=1}^{\infty} (-1)^{n-1} n$

c. $\sum_{n=1}^{\infty} \frac{n^{16}}{6^n}$

d. $\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$