MATH 253 (PHILLIPS) MIDTERM 0 VERSION 1

INSTRUCTIONS: No books, notes, or calculators are permitted on this test. Write all answers in the spaces provided at the right, except for Problem 10, which requires you to show correct reasoning. Do scratchwork on the back or on blank paper provided for this purpose. No partial credit, except a few very minor errors (for example, omitting a constant of integration) will lose only half the credit. Time: 30 minutes.

1. Write as a single fraction, and simplify as much as possible: \( \frac{2}{a + 1} - \frac{2}{a - 2} \)

Answer: 

2. Simplify the following expression as much as possible. If no simplification is possible, write “not possible”: \( \frac{2 \sin(5x) - 5}{2 \sin(5x) + 5} \)

Answer: 

3. Let \( f(x) = 2 - x \). Evaluate the expression \( f(x + 3) - f(2x + 1) \), and simplify it as much as possible.

Answer: 

4. Find all real solutions to the equation \( 8y^{-1} = -15y^{-2} - 1 \). If no real solution exists, write “no solution”.

Answer: 

5. Find all real numbers \( a \) such that \((-a, 7)\) is in the first quadrant (and not on any of the coordinate axes).

Answer: 

6. Find all real numbers \( x \) such that \(|x - 7| \leq 2\).

Answer: 

7. Let \( h(x) = \sin(x^3 - 31 \tan(x)) \). Find \( h'(x) \).

Answer: 

8. Find \( \int (1 - 3t)^{100} dt \).

Answer: 

9. Find \( \frac{d}{dx} \left( \int_{\sqrt{x}}^{x} \cos(2t^2) dt \right) \).

Answer: 

10. Determine whether the improper integral \( \int_{1}^{\infty} \frac{8}{1 + x^4} \, dx \) converges. Show your work (below or on the back side); it must be correct to get credit for this problem. (No partial credit!) You need not actually evaluate the integral.