1. Write as a single fraction, and simplify as much as possible: \[ \frac{1}{y-2} - \frac{1}{y+2} \]

Solution:
\[
\frac{1}{y-2} - \frac{1}{y+2} = \frac{y+2}{(y-2)(y+2)} - \frac{y-2}{(y-2)(y+2)} = \frac{4}{(y-2)(y+2)}.
\]

2. Simplify the following expression as much as possible. If no simplification is possible, write “not possible”: \[ \frac{x^2 + 3x}{x^2 + 6x} \]

Solution:
\[
\frac{x^2 + 3x}{x^2 + 6x} = \frac{x(x+3)}{x(x+6)} = \frac{x+3}{x+6}.
\]

The last expression can’t be further simplified.

3. Let \( f(x) = 7 - x \). Evaluate the expression \( f(x+3) - f(x-2) \), and simplify it as much as possible.

Solution:
\[
f(x+3) - f(x-2) = 7 - (x+3) - (7-(x-2)) = 7-x-3-(7-x+2) = 7-x-3-7+x-2 = -5.
\]

4. Find all real solutions to the equation \( \frac{6}{x} + \frac{7}{x^2} = 1 \). If no real solution exists, write “no solution”.

Solution: Multiply through by \( x^2 \), getting:
\[
6x + 7 = x^2 \\
x^2 - 6x - 7 = 0 \\
(x + 1)(x - 7) = 0 \\
x = -1 \text{ or } x = 7.
\]

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by \( x^2 \) at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

The equation can also be solved by treating it as a quadratic equation in \( 1/x \).

5. Find all real numbers \( x \) such that \( |x + 2| < 1 \).

Solution:
\[
|x + 2| < 1 \\
-1 < x + 2 < 1 \\
-3 < x < -1.
\]

The last step was done by subtracting 2 everywhere. So the answer is \( -3 < x < -1 \). One can also see this directly, using the interpretation of the absolute value as a distance: \( x \) is at distance less than 1 from \(-2\), so \( -3 < x < -1 \).

(Continued on back or next page)
6. Find all real numbers \(a\) such that \(|a| = -a\).

Solution: \(|a| = -a\) if and only if \(a \leq 0\).

7. Let \(h(x) = (2x^{15} - \arcsin(x))^9\). Find \(h'(x)\).

Solution: We use the chain rule, getting

\[
h'(x) = 9(2x^{15} - \arcsin(x))^8 \cdot \frac{d}{dx}(2x^{15} - \arcsin(x))
\]

\[
= 9(2x^{15} - \arcsin(x))^8 \left( 2 \cdot 15x^{14} - \frac{1}{\sqrt{1-x^2}} \right)
\]

\[
= 9(2x^{15} - \arcsin(x))^8 \left( 30x^{14} - \frac{1}{\sqrt{1-x^2}} \right).
\]

The simplification in the last step is required.

8. Find \(\int e^{1-2y} \, dy\).

Solution: Use the substitution \(w = 1 - 2y\), so \(dy = -\frac{1}{2} \, dw\), to get

\[
\int e^{1-2y} \, dy = -\int \frac{1}{2} e^w \, dw = -\frac{1}{2} e^w + C = -\frac{1}{2} e^{1-2y} + C.
\]

(You should be able to do this correctly without writing down the substitution explicitly.)

9. Find \(\frac{d}{dx} \left( \int_{7}^{x} e^{17+t^5} \, dt \right)\).

Solution: \(\frac{d}{dx} \left( \int_{7}^{x} e^{17+t^5} \, dt \right) = e^{17+x^5}\) by the Fundamental Theorem of Calculus. (The answer must be a function of \(x\), so that \(e^{17+x^5}\) is wrong, and gets no credit.)

10. Determine whether the improper integral \(\int_{1}^{\infty} \frac{1}{5\sqrt{x} - 1} \, dx\) converges. Show your work (below or on the back side); it must be correct to get credit for this problem. (No partial credit!) You need not actually evaluate the integral.

Solution: Use the comparison test for improper integrals. We have \(\frac{1}{5\sqrt{x} - 1} \geq \left( \frac{1}{5} \right) \left( \frac{1}{\sqrt{x}} \right)\) for \(x \geq 1\). Now \(\frac{1}{5}\) is a positive constant, and we know that \(\int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx\) diverges, so \(\int_{1}^{\infty} \left( \frac{1}{5} \right) \left( \frac{1}{\sqrt{x}} \right) \, dx\) must diverge. The comparison test now implies that \(\int_{1}^{\infty} \frac{1}{5\sqrt{x} - 1} \, dx\) also diverges.

You must get the inequality in the right direction! It is also true that \(\frac{1}{5\sqrt{x} - 1} \leq \frac{1}{\sqrt{x}}\) for \(x \geq 1\). This, however, is useless: since \(\int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx\) diverges, this inequality tells you nothing about \(\int_{1}^{\infty} \frac{1}{5\sqrt{x} - 1} \, dx\).