GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

2. Closed book, except for a $3 \times 5$ file card.

3. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.

4. The point values are as indicated in each problem; total 100 points.

5. Write all answers on the test paper. Use the space below the extra credit problems for long answers or scratch work.

6. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit. Showing work includes saying why any convergence test or error estimate you use actually applies.

7. Unless otherwise specified, you can use the series expansions centered at $x = 0$ for $e^x$, $\sin(x)$, $\cos(x)$, $\arctan(x)$, $\ln(1 + x)$, and $(1 + x)^k$ (binomial series) without deriving them.

8. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.

9. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.

10. Final answers must always be simplified unless otherwise specified. (General principle: Combinations of powers and factorials need not be multiplied out, and “special” cancellations need not be made. Thus, in most problems, the expression $3^7/7!$ is fine. However, $(n + 2)!/n!$ should be simplified to $(n + 1)(n + 2)$.)

11. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).

12. Time: 50 minutes, unless extended by an early start.

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1. (1 point) Are you awake?

2. (10 points.) Determine whether or not the series \( \sum_{n=0}^{\infty} \frac{\sin(n^2)}{4^n + n} \) is convergent. For any convergence test that you use, be sure to say why it applies.

3. (10 points.) You want to approximate \( \sum_{n=0}^{\infty} \frac{(-1)^n}{1 + n^2} \) to within 0.01. Which partial sum should you use? Why? Make sure to explain why the hypotheses of any estimate you use are satisfied.
4. (10 points.) Determine whether or not the series \( \sum_{n=0}^{\infty} \frac{2^n(n!)^2}{(2n)!} \) is convergent. For any convergence test that you use, be sure to say why it applies.

5. (10 points.) Use Taylor’s Inequality (the remainder estimate) to give an upper bound on the error that results from approximating the function \( f(x) = \ln(x) \) by its Taylor polynomial of degree 3 centered at 5 on the interval \([3, 7]\).
6. (10 points.) Define a function $S$ by

$$S(x) = \begin{cases} \frac{\sin(2x)}{x} & x \neq 0 \\ 2 & x = 0. \end{cases}$$

Find $S''(0)$. Give justification.

7. (18 points.) Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x - 3)^n}{4^{n+2} \sqrt{n}}.$$ For any convergence test that you use, be sure to say why it applies.
8. (15 points.) Find the Taylor series centered at 0 for the function \( f(x) = \int_0^x e^t \, dt \). (You need not proceed directly from the definition of the Taylor series.) Then find its radius of convergence. For any convergence test that you use, be sure to say why it applies.

9. (16 points.) Determine whether the series \( \sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n} \) is convergent, and determine whether it is absolutely convergent. For any convergence test that you use, be sure to say why it applies.

Extra credit on back of page.
Extra credit. (Do not attempt this problem until you have done and checked your answer to all the ordinary problems on this exam. It will only be counted if you get a grade of B or better on the main part of this exam.)

EC1. (15 extra credit points.) Find the radius of convergence and the interval of convergence of the series \( \sum_{n=0}^{\infty} e^{\sin(n)} x^n \). For any convergence test that you use, be sure to say why it applies.

EC2. (15 extra credit points.) Find, with justification, a number \( N \) such that the \( N \)-th partial sum of the series \( \sum_{n=0}^{\infty} e^{-n^2} \) is within \( 10^{-10} \) of the sum of the series.