1. (1 point) Are you awake?

2. (10 points.) Determine whether or not the series \( \sum_{n=0}^{\infty} \frac{\sin(n^2)}{4^n + n} \) is convergent. For any convergence test that you use, be sure to say why it applies.

3. (10 points.) You want to approximate \( \sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2} \) to within 0.01. Which partial sum should you use? Why? Make sure to explain why the hypotheses of any estimate you use are satisfied.

4. (10 points.) Determine whether or not the series \( \sum_{n=0}^{\infty} \frac{2^n(n!)^2}{(2n)!} \) is convergent. For any convergence test that you use, be sure to say why it applies.

5. (10 points.) Use Taylor’s Inequality (the remainder estimate) to give an upper bound on the error that results from approximating the function \( f(x) = \ln(x) \) by its Taylor polynomial of degree 3 centered at 5 on the interval \([3, 7]\).

6. (10 points.) Define a function \( S \) by

\[
S(x) = \begin{cases} 
\frac{\sin(2x)}{x} & x \neq 0 \\
2 & x = 0.
\end{cases}
\]

Find \( S''(0) \). Give justification.

7. (18 points.) Find the radius of convergence and the interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(x - 3)^n}{4^n + 2 \sqrt{n}} \). For any convergence test that you use, be sure to say why it applies.

8. (15 points.) Find the Taylor series centered at 0 for the function \( f(x) = \int_0^x e^{t^2} \, dt \). (You need not proceed directly from the definition of the Taylor series.) Then find its radius of convergence. For any convergence test that you use, be sure to say why it applies.
9. (16 points.) Determine whether the series \( \sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n} \) is convergent, and determine whether it is absolutely convergent. For any convergence test that you use, be sure to say why it applies.

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.)

EC1. (15 extra credit points.) Find the radius of convergence and the interval of convergence of the series \( \sum_{n=0}^{\infty} e^{\sin(n)} x^n \). For any convergence test that you use, be sure to say why it applies.

EC2. (15 extra credit points.) Find, with justification, a number \( N \) such that the \( N \)-th partial sum of the series \( \sum_{n=0}^{\infty} e^{-n^2} \) is within \( 10^{-10} \) of the sum of the series.