Recall the binomial series, for $(1 + x)^r$, for any real number $r$:

\[ \sum_{n=0}^{\infty} \binom{r}{n} x^n = \sum_{n=0}^{\infty} \frac{r(r-1)(r-2)\cdots(r-n+1)}{n!} x^n. \]

Also recall the degree 6 Taylor polynomial for $f(x) = (x + 1)^{3/2}$, from Friday’s worksheet:

\[ p_6(x) = 1 + \left( \frac{3}{2} \right) x + \left( \frac{3 \cdot 1}{2^2 \cdot 2!} \right) x^2 + \left( \frac{3 \cdot 1 \cdot (-1)}{2^3 \cdot 3!} \right) x^3 \\
+ \left( \frac{3 \cdot 1 \cdot (-1) \cdot (-3)}{2^4 \cdot 4!} \right) x^4 + \left( \frac{3 \cdot 1 \cdot (-1) \cdot (-3) \cdot (-5)}{2^5 \cdot 5!} \right) x^5 \\
+ \left( \frac{3 \cdot 1 \cdot (-1) \cdot (-3) \cdot (-5) \cdot (-7)}{2^6 \cdot 6!} \right) x^6. \]

1. Consider $g(x) = (x + 1)^{4/3}$. Write the degree 6 Taylor polynomial for this function in the same manner as was done above for $f(x) = (x + 1)^{3/2}$.

2. A sequence $(c_n)_{n=0}^{\infty}$ starts with $c_0 = \frac{8}{9}$, $c_1 = \frac{4}{3}$, $c_2 = 2$, $c_3 = 3$, and $c_4 = \frac{9}{2}$. Assuming the pattern continues, what is $c_5$ and what is the formula for $c_n$ for general $n$?

3. (If time.) A sequence $(a_n)_{n=1}^{\infty}$ is given by $a_n = \frac{(n-1)!}{3 \cdot 2^n}$ for strictly positive integers $n$. Write the first four terms $a_1$, $a_2$, $a_3$, and $a_4$ as ratios of integers, first without cancelling common factors, and then after cancelling common factors. (Use the back of the page, or a separate page.)

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