1. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-2)^n}{7^n}$ is convergent or divergent. If it is convergent, find the sum.

2. Determine whether the series $\sum_{n=3}^{\infty} \frac{(-2)^n}{7^{n+5}}$ is convergent or divergent. If it is convergent, find the sum.

(Why does convergence have the same answer as in Problem 1? If the series in Problem 1 converges, what factor do you need to multiply its sum by to get the sum of this one?)

3. Consider the two telescoping series

$$\sum_{n=1}^{\infty} [\sqrt{n} - \sqrt{n+1}] \quad \text{and} \quad \sum_{n=1}^{\infty} \left( \sqrt{3 + \frac{17}{n}} - \sqrt{3 + \frac{17}{n+1}} \right).$$

One of these converges, and one diverges. Which is which, and why?

4. On the back, use the Integral Test to show that $\sum_{n=1}^{\infty} \frac{1}{n^8}$ converges.

Be sure to check that all the hypotheses of the Integral Test actually hold! Homework and exam problems in which this is not done will lose substantial credit.

Date: 24 April 2023.