WORKSHEET: COMPARISON TEST

Comparison test for convergence of series:

1. If \( 0 \leq a_n \leq b_n \) for \( n = 1, 2, 3, \ldots \), and \( \sum_{n=1}^{\infty} b_n \) is convergent, then \( \sum_{n=1}^{\infty} a_n \) is convergent.

2. If \( 0 \leq a_n \leq b_n \) for \( n = 1, 2, 3, \ldots \), and \( \sum_{n=1}^{\infty} a_n \) is divergent, then \( \sum_{n=1}^{\infty} b_n \) is divergent.

It is essential to get the inequalities in the right direction. For example, if \( 0 \leq a_n \leq b_n \) for \( n = 1, 2, 3, \ldots \), and \( \sum_{n=1}^{\infty} a_n \) is convergent, this tells you nothing about \( \sum_{n=1}^{\infty} b_n \).

1. Determine whether or not the series \( \sum_{n=0}^{\infty} \frac{1}{3^n + 2n} \) is convergent. Be sure to show your reasoning.

Hint: The series \( \sum_{n=0}^{\infty} \frac{1}{3^n} \) is convergent.

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2. Determine whether or not the series \( \sum_{n=1}^{\infty} \frac{5 + 3 \sin(n)}{\sqrt{n}} \) is convergent. Be sure to show your reasoning.

Hint: The series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \) is divergent.

3. Determine whether or not the series \( \sum_{n=1}^{\infty} \frac{\arctan(n^3)}{n^3} \) is convergent. Be sure to show your reasoning.

Hint: The series \( \sum_{n=1}^{\infty} \frac{1}{n^3} \) is convergent.