1. Write as a single fraction, and simplify as much as possible: \( \frac{3}{b+3} - \frac{1}{b+5} \)

Solution:

\[
\frac{3}{b+3} - \frac{1}{b+5} = \frac{3(b + 5) - (b + 3)}{(b + 3)(b + 5)} = \frac{3b + 15 - b - 3}{(b + 3)(b + 5)} = \frac{2b + 12}{(b + 3)(b + 5)}. 
\]

2. Simplify the following expression as much as possible. If no simplification is possible, write “not possible”: \( \frac{\sin(2y)}{\sin(2y) + 2} \)

Solution: The expression \( \frac{\sin(2y)}{\sin(2y) + 2} \) can’t be simplified.

3. Let \( f(x) = 7 - x \). Evaluate the expression \( f(17) - f(2x - 3) \), and simplify it as much as possible.

Solution:

\[
f(17) - f(2x - 3) = 7 - 17 - (7 - (2x - 3)) = 7 - 17 - 7 + 2x - 3 = 2x - 20. 
\]

4. Simplify completely (for \( y > 0 \)): \( \frac{\left( \frac{2}{\sqrt{y}} \right)}{\left( \frac{y^{3/2}}{6} \right)} \)

Solution:

\[
\frac{\left( \frac{2}{\sqrt{y}} \right)}{\left( \frac{y^{3/2}}{6} \right)} = \left( \frac{2}{\sqrt{y}} \right) \left( \frac{6}{y^{3/2}} \right) = \frac{2 \cdot 6}{\sqrt{y}y^{3/2}} = \frac{4}{3y^2}. 
\]

If you want, you can also write the answer as \( \frac{4}{3y^{-2}} \), but this is not necessary.

5. Find all real numbers \( x \) such that \( |x + 6| \leq 2 \).

Solution:

\[
|x + 6| \leq 2 \\
-2 \leq x + 6 \leq 2 \\
-8 \leq x \leq -4 
\]

The last step was done by subtracting 6 everywhere. So the answer is \(-8 \leq x \leq -4\). One can also see this directly, using the interpretation of the absolute value as a distance: \( x \) is at distance less than 2 from \(-6\), so \(-8 \leq x \leq -4\).

(Continued on back or next page)
6. Find all real numbers $a$ such that $(2a, -a)$ is in the second quadrant (and not on any of the coordinate axes).

Solution: The point $(2a, -a)$ is in the second quadrant if and only if both $2a < 0$ and $-a > 0$, which happens if and only if $a < 0$.

7. Let $f(x) = (6x^{11} + \ln(x))^4$. Find $f'(x)$.

Solution: We use the chain rule, getting

$$f'(x) = 4(6x^{11} + \ln(x))^3 \cdot \frac{d}{dx}(6x^{11} + \ln(x))$$

$$= 4(6x^{11} + \ln(x))^3 \left(6 \cdot 11x^{10} + \frac{1}{x}\right) = 4(6x^{11} + \ln(x))^3 \left(66x^{10} + \frac{1}{x}\right).$$

The simplification in the last step is required.

8. Find $\int \sin(2 - 5x) \, dx$.

Solution: Use the substitution $u = 2 - 5x$, so $dx = -\frac{1}{5} \, du$, to get

$$\int \sin(2 - 5x) \, dx = \int \sin(u) \left(-\frac{1}{5}\right) \, du = -\cos(u) \left(-\frac{1}{5}\right) + C = \frac{1}{5} \cos(2 - 5x) + C.$$  
(You should be able to do this correctly without writing down the substitution explicitly.)

9. Find $\frac{d}{dx} \left(\int_9^x \cos(3t + t^5) \, dt\right)$.

Solution:

$$\frac{d}{dx} \left(\int_9^x \cos(3t + t^5) \, dt\right) = \cos(3x + x^5)$$

by the Fundamental Theorem of Calculus. (The answer must be a function of $x$, so that $\cos(3t + t^5)$ is wrong, and gets no credit.)

10. Determine whether the improper integral $\int_3^\infty \frac{3}{7x^{5/7} - 2} \, dx$ converges. Show your work (below or on the back side); it must be correct to get credit for this problem. (No partial credit!) You need not actually evaluate the integral.

Solution: Use the comparison test for improper integrals. We have

$$\frac{3}{7x^{5/7} - 2} \geq \left(\frac{3}{7}\right) \left(\frac{1}{x^{5/7}}\right) > 0$$

for $x \geq 3$. Now $\frac{3}{7}$ is a positive constant, and we know that $\int_3^\infty \frac{1}{x^{5/7}} \, dx$ diverges, so

$$\int_3^\infty \left(\frac{3}{7}\right) \left(\frac{1}{x^{5/7}}\right) \, dx$$

must diverge. The comparison test now implies that $\int_3^\infty \frac{3}{7x^{5/7} - 2} \, dx$ also diverges.

You must get the inequality in the right direction! It is also true that $\frac{3}{7x^{5/7} - 2} \leq \frac{3}{5x^{5/7}}$ for $x \geq 3$. This, however, is useless: since $\int_3^\infty \frac{1}{5x^{5/7}} \, dx$ diverges, this inequality tells you nothing about $\int_3^\infty \frac{3}{7x^{5/7} - 2} \, dx$. 