1. (1 point) True or false: The Alternating Series Test was included in the course for the purpose of confusing students who are having trouble remembering the relation between convergence of a series and whether its summands go to zero.

2. (12 points) Determine whether or not the series \( \sum_{n=0}^{\infty} \frac{\sin(3^n)}{3^n + \sqrt{n}} \) is convergent. Be sure to show your reasoning in mathematically and notationally correct steps. In particular, for any convergence test that you use, be sure to demonstrate why it applies.

3. (12 points) Determine whether or not the series \( \sum_{n=1}^{\infty} \frac{1 + 4 \sin^2(n)}{\sqrt{n}} \) is convergent. Be sure to show your reasoning in mathematically and notationally correct steps. In particular, for any convergence test that you use, be sure to demonstrate why it applies.

4. (12 points) Find the radius of convergence of the series \( \sum_{n=1}^{\infty} \frac{n(x + 7)^n}{3^{2n}} \). (You need not find the interval of convergence.) Be sure to show your reasoning in mathematically and notationally correct steps. In particular, for any convergence test that you use, be sure to demonstrate why it applies.

5. (16 points) Consider the power series \( \sum_{n=1}^{\infty} \frac{(-1)^n(x - 5)^n}{8^n \sqrt{n + 1}} \). You are told that its radius of convergence is 8. Given this, find its interval of convergence. Be sure to show your reasoning in mathematically and notationally correct steps. In particular, for any convergence test that you use, be sure to demonstrate why it applies.

6. (12 points) Find, with justification, some number \( n \) such that the approximation of \( f(x) = e^x \) by its Taylor polynomial of degree \( n \) centered at 0 gives an error of less than 0.0001 when \( x = \frac{1}{20} \). You need not find the smallest choice of \( n \). Be sure to demonstrate, using mathematically and notationally correct steps, that your choice of \( n \) actually works.
7. (12 points.) Find the Taylor series centered at 0 for the function \( f(x) = \int_0^x \sin(t^4) \, dt \). (You need not proceed directly from the definition of the Taylor series.) For which values of \( x \) does your series actually converge to \( f(x) \)? Be sure to show your reasoning in mathematically and notationally correct steps.

8. (12 points.) Give an upper bound on the error that results from approximating \( \sum_{k=1}^{\infty} \frac{1}{k^6} \) using the partial sum \( s_{10} = \sum_{k=1}^{10} \frac{1}{k^6} \). Give a mathematically and notationally correct justification for your answer. In particular, be sure to explain why the hypotheses for any estimate you use are satisfied.

9. (11 points.) Determine whether the series \( \sum_{n=1}^{\infty} \frac{(-1)^n+1(7n+2)^n}{(6n+135)^n} \) converges. Be sure to show your reasoning in mathematically and notationally correct steps. In particular, for any convergence test that you use, be sure to demonstrate why it applies.

Extra credit. (15 extra credit points.) For the series \( \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \), give, with proof, in mathematically and notationally correct steps, an integer \( n \) such that the \( n \)-th partial sum of this series is within \( 10^{-6} \) of the true sum. In particular, explain why the hypotheses of any estimate you use are satisfied. You need not find the best value of \( n \).

Hint: None of the methods for error estimates presented in this course apply directly, but a combination of several ideas from this course will give a short argument.