1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.


3. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.

4. The point values are as indicated in each problem; total 100 points.

5. Write all answers on the test paper.

6. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.

7. Unless otherwise specified, you can use the series expansions centered at $x = 0$ for $e^x$, $\sin(x)$, $\cos(x)$, and $(1 + x)^k$ (binomial series) without deriving them. You can also use the standard facts about geometric series and the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

8. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.

9. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.

10. Final answers must always be simplified unless otherwise specified. (General principle: Combinations of powers and factorials need not be multiplied out, and “special” cancellations need not be made. Thus, in most problems, the expression $3^7/7!$ is fine. However, $(n+2)!/n!$ should be simplified to $(n+1)(n+2)$.

11. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).

12. Time: 50 minutes, unless extended by an early start.

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1. (7 points/part.) For each of the following sequences \( (a_n)_{n=1}^\infty \), find its limit (possibly \( \infty \) or \( -\infty \)), giving reasons, or explain why the sequence neither converges nor diverges to \( \infty \) or \( -\infty \).

   a. \( a_n = \frac{\sqrt[n]{n} + 55}{55 \sqrt[n]{n} + 11} \) for strictly positive integers \( n \).

   b. \( a_n = \frac{(3n)!}{(3n + 1)!} \) for nonnegative integers \( n \).

   c. \( (a_n)_{n=1}^\infty = \left( -16, 12, -9, \frac{27}{4}, -\frac{81}{16}, \ldots \right) \).
2. (8 points/part.) For each of the following functions, find the Taylor polynomial of the given degree and centered at the given point. (Don’t multiply out the coefficients.)

a. \( f(x) = (x^5 + 2) \sin(x) \), degree 8, centered at \( x = 0 \).

b. \( f(x) = \cos(x) \), degree 7, centered at \( x = \frac{\pi}{2} \) (not \( x = 0 \)).

c. \( f(x) = \frac{2e^x}{1 - \sin(x)} \), degree 3, centered at \( x = 0 \).
3. (8 points/part.) For each of the following series, determine whether it is convergent. Be sure to show your reasoning. If the series is convergent, find its sum.

a. \( \sum_{n=0}^{\infty} \frac{2^n}{3^{n-2}} \).

b. \( \sum_{n=1}^{\infty} \left( \frac{1}{\ln(n + 1)} - \frac{1}{\ln(n + 3)} \right) \).

c. \( \sum_{n=1}^{\infty} e^{1/n^4} \).
4. (8 points.) Use the Integral Test to determine whether the series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{4n + 119}} \) is convergent. Be sure to show your reasoning.

5. (8 points.) Determine whether or not the series \( \sum_{n=1}^{\infty} \frac{1}{3n^2 + 2 \arctan(n)} \) is convergent. Be sure to show your reasoning.
6. (7 points.) Let \((a_n)^\infty_{n=1}\) be a sequence, and let \(L\) be a real number. State the precise definition of what it means to have \(\lim_{n \to \infty} a_n = L\).

7. (8 points.) Define a sequence \((x_n)^\infty_{n=1}\) by \(x_n = \frac{4n + 7}{n}\) for \(n = 1, 2, \ldots\). For \(\varepsilon = 0.1\), find some integer \(N > 0\) such that for all \(n > N\) we have \(|x_n - 4| < \varepsilon\), and show that your choice works. (You need not find the best value of \(N\).)

Extra credit. (12 extra credit points. Do not attempt this problem until you have done and checked your answer to all the ordinary problems on this exam. It will only be counted if you get a grade of B or better on the main part of this exam.)

Prove directly from the definition that \(\lim_{n \to \infty} \left(\frac{n}{2} - 19\right) = \infty\).