Math 263

Sample Questions for the Midterm

Problems.

1. (12 points.) Prove directly from the definition that \( \lim_{n \to \infty} \frac{2n + 3}{7n - 100} = \frac{2}{7} \).

2. (8 points) Determine, with proof, whether or not the series \( \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 2} \) is convergent.

3. (8 points.) Determine, with proof, whether or not the series \( \sum_{n=1}^{\infty} \frac{n! \cdot n^2}{(n + 4)!} \) is convergent.

4. (8 points.) Determine, with proof, whether or not the series \( \sum_{n=1}^{\infty} \frac{e^{\sin(n)}}{n} \) is convergent.

5. (8 points.) Determine whether or not the series \( \sum_{n=1}^{\infty} \frac{3 + 7\sin^6(n)}{n^{7/5}} \) is convergent. For any convergence test that you use, be sure to say why it applies.

6. (12 points.) Prove directly from the definition that \( \lim_{n \to \infty} \frac{6}{\sqrt{n^2 + 3\sin(n^2)}} = 0 \).