MATH 281 (PHILLIPS): FINAL EXAM PROBLEM LIST

Problem 1 (1 point)**.** You have stolen a time machine. You are going back in time to assassinate which of these people:
(a) The inventor of cross products.
(b) The inventor of the osculating circle to a curve.
(c) The inventor of Lagrange multipliers.
(d) The inventor of `WeBWorK`.

Problem 2 (18 points)**.** Find an equation for the tangent plane to the surface

$$xy + \sin(xz) + 3z = y^3 + 5$$

at the point $(0, 1, 2)$.

Problem 3 (12 points)**.** Let k be a function such that $k'(t) = \cos(\sin(t))$. Suppose

$$f(x, y, z) = e^{2x} + \arctan(z)k(xy^2 - x^2y) - \ln(15).$$

Find $D_2 f(x, y, z)$.

Problem 4 (20 points)**.** Find and classify (as local maximums, local minimums, or saddle points) all critical points of the function $f(x, y) = 3x - x^3 - 3xy^2$.

Problem 5 (10 points)**.** Let f, x, and y be differentiable functions of two variables, and let $H(s, t) = f(x(s, t),\ y(s, t))$. Suppose that

$$\begin{gathered} x(5,2) = 2, \qquad D_1 x(5,2) = -3, \qquad D_2 x(5,2) = 5, \\ y(5,2) = 5, \qquad D_1 y(5,2) = -4, \qquad D_2 y(5,2) = 2, \\ f(5,2) = -11, \qquad D_1 f(5,2) = 4, \qquad D_2 f(5,2) = 3, \\ f(2,5) = -1, \qquad D_1 f(2,5) = -2, \qquad \text{and} \qquad D_2 f(2,5) = 7. \end{gathered}$$

Find $D_2 H(5, 2)$.

Problem 6 (15 points)**.** Find an equation for the normal plane to the parametric curve given by $\mathbf{r}(t) = \big\langle 4\sin(2t - 4) + 3,\ t,\ t^2 - 5t \big\rangle$ at the point $(3,\ 2,\ -6)$.

Problem 7 (20 points)**.** Find the maximum and minimum values of $x - 3y$ on the circle $x^2 + y^2 = 10$.

Problem 8 (14 points)**.** At position (x, y, z), measured in meters, and if $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, and $0 \leq z \leq 10$, the relative humidity of the air in percentage points is given by

$$H(x, y, z) = 58 + \frac{1}{2000}\big(x^3 + 2xy^2 - 3z\big).$$

A mosquito, looking for an animal to bite, is located at $(-2, 3, 5)$. If it flies in a straight line towards the point $(0, -3, 2)$ at one meter per minute, at what rate does the relative humidity of the air it is in change? Be sure to include units.

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Problem 9 (12 points)**.** A parametrized curve is give by $\mathbf{r}(t) = \langle t,\ 15,\ t^3\rangle$. Find its curvature at time t.

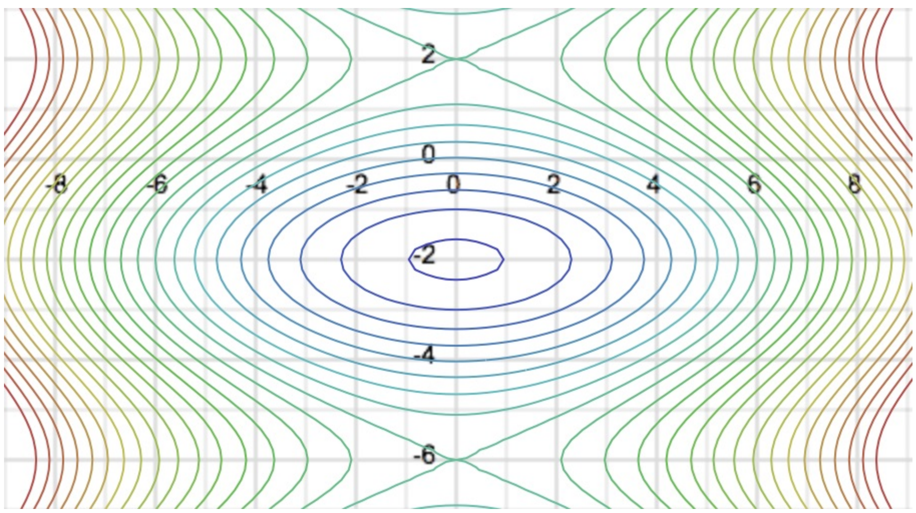
Problem 10 (25 points)**.** Find the maximum and minimum values of the function

$$f(x, y) = (x - y)^2 + (x - 4)^2 - 1$$

on the closed rectangle with vertices $(0, 0)$, $(0, 2)$, $(4, 0)$, and $(4, 2)$.

Problem 11 (12 points)**.** Find an equation for the plane that contains the line $x = 1 - 2t$, $y = 3 + 2t$, $z = t - 1$ and contains the point $(0, 1, 2)$.

Problem 12 (3 points/part)**.** The picture below is a partial of a contour plot of a function $z = T(x, y)$. The contour lines are evenly spaced, with the darkest red (at the right and left) being at the value 9.8 and the darkest blue (in the middle) being at the value -1.9.



For each of the following questions, give an answer and (this is important) a brief justification.

(1) Is $D_1T(-6,\ -2)$ near zero, clearly positive, or clearly negative?
(2) Is $D_2T(-6,\ -2)$ near zero, clearly positive, or clearly negative?
(3) Is $(-6,\ -2)$ close to being a local minimum, a local maximum, a saddle point, or none of these?
(4) Is $(0,\ -2)$ close to being a local minimum, a local maximum, a saddle point, or none of these?
(5) Give a unit vector $\mathbf{u}$ which points roughly in the same direction as $\mathbf{grad}(T)(6,\ -4)$.

Problem 13 (14 points)**.** A drone has acceleration $\mathbf{a}(t) = 2\mathbf{j} + 8\cos(2t)\mathbf{k}$. If it has initial velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$ and initial position $\mathbf{r}(0) = 2\mathbf{i} + 12\mathbf{k}$, find its velocity and position vectors.

Problem 14 (12 points)**.** Find $\lim_{(x,y)\to(0,0)} x^2 \sin\left(\dfrac{x}{x^2 + y^2}\right)$, giving reasons for your answer, or explain why it does not exist.

Problem 15 (25 extra credit points)**.** Find the maximum and minimum values of the function $f(x, y, z) = 3x^2 + y^2 - y - z^2$ on the solid cylinder bounded by $x^2 + z^2 = 4$, $y = 3$, and $y = -1$. Explain the reasons for your procedure.