FINAL EXAM SAMPLE PROBLEMS

The final exam is cumulative, but will give higher weight to material not tested before, that is, after the second midterm. (Because of the short period after the second midterm, this will still probably be less than half the exam, perhaps substantially less.)

Most problems will be similar to `WeBWorK` problems, written homework problems, problems from the real and sample midterms, and the problems here. Note, though, that the exact form of functions which appear could vary substantially. In particular, a problem requiring computation of the limit or derivative of a single variable function could be modified to use any other single variable function; the methods required to find the limit or derivative could be very different. (This includes computations of partial derivatives, which are really just derivatives of single variable functions.)

Be sure to get the notation right! (This is a frequent source of errors.) The right notation will help you get the mathematics right, and incorrect notation will lose points even if there are no other errors.

The problems have point values attached, which give a rough idea of the point values problems requiring a similar amount of work will have on the real exam. The real final will total 200 points, and the problems here total much more than that.

This problem list mostly does not repeat earlier problem types. It contains mostly problems on material after Midterm 2 and problems of types on previous material which should have been in previous sample problem lists but weren't.

Problem 1 (8 points)**.** You want to use Lagrange multipliers to minimize the quantity ye^{x-z} subject to the constraints $9x^2 + 4y^2 + 36z^2 = 36$ and $xy = 1 - yz$. Set up, but do **not** attempt to solve, equations for the points (x, y, z) at which to examine the values of f.

Problem 2 (18 points)**.** Use Lagrange multipliers to find the maximum and minimum values of $2x^2 + 3y^2 - 4x - 5$ on the circle $x^2 + y^2 = 16$.

Problem 3 (8 points)**.** You want to use Lagrange multipliers to maximize the quantity $x^4 + y^6 + z^8$ subject to the constraint $2x^2 + 4y^2 = 65 - 6z^2$. Set up, bu do **not** attempt to solve, equations for the points (x, y, z) at which to examine the values of f.

Problem 4 (8 points)**.** Set $f(x, y, z) = x \sin(yz^2)$. Find the directional derivative of f at the point $(1, 3, 0)$ in the (unit length) direction determined by the vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$.

Problem 5 (14 points)**.** On the planet Yuggxth, a ball is thrown into the air from the origin towards the east (positive x direction), with initial velocity $50\mathbf{i} + 80\mathbf{k}$ (with speed measured in feet per second). Because of spin on the ball, it experiences a southwards acceleration of $4\,\text{feet/second}^2$. The acceleration due to gravity at the surface of the planet Yuggxth is exactly $20\,\text{feet/second}^2$. Where does the ball hit the ground, and with what speed?

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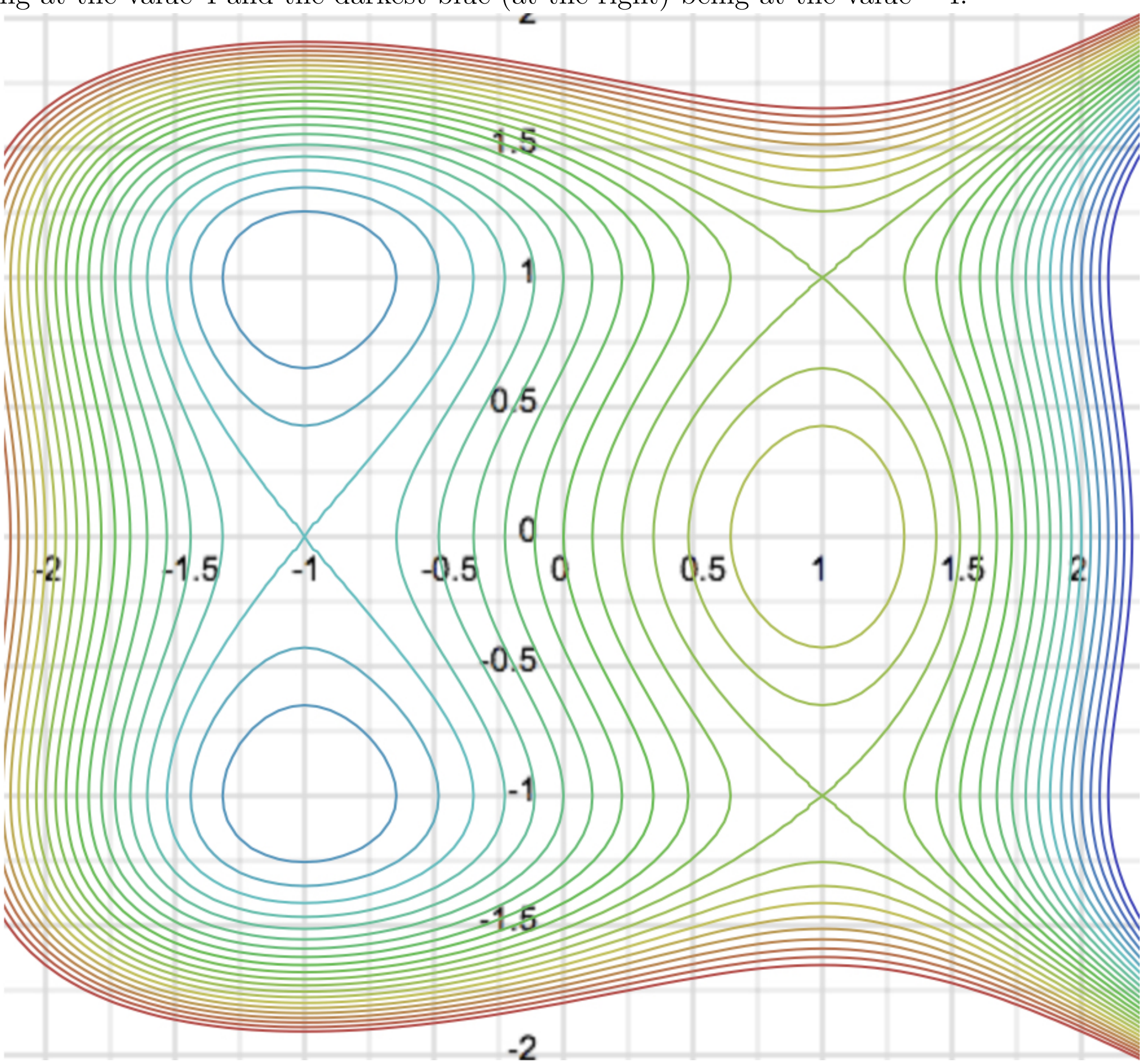
Problem 6 (12 points)**.** Find $\lim_{(x,y)\to(0,0)}(x^2+y^4)\cos\left(\frac{7}{2x^6+y^2}\right)$, giving reasons for your answer, or explain why it does not exist.

Problem 7 (10 points)**.** The picture below is a partial of a contour plot of a function $z = Q(x,y)$. The contour lines are evenly spaced, with the darkest red (at the top and bottom) being at the value 4 and the darkest blue (at the right) being at the value -4.



Identify all critical points in the region shown, and for each one say whether it is a local minimum, local maximum, saddle point, or none of these. Give reasons.

Problem 8 (8 points)**.** A function f of two variables satisfies $f(2,7) = -3$, $f_x(2,7) = 0$, $f_y(2,7) = 0$, $f_{xx}(2,7) = -4$, $f_{yy}(2,7) = -5$, and $f_{xy}(2,7) = 2$. Does f have a local minimum at $(2,7)$, a local maximum at $(2,7)$, a saddle point at $(2,7)$, none of these, or is it impossible to determine? Why?

Problem 9 (15 points)**.** For the function $f(x,y) = 4y^3 - 3y + 12x^2y$, find all critical points and identify each as a local maximum, local minimum, or saddle point.

Problem 10 (10 points)**.** Suppose the position of a particle at time t is given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$. Find the tangential and normal components of its acceleration vector.

Problem 11 (10 points)**.** Find the velocity and position vectors of a particle that has acceleration $\mathbf{a}(t) = -t\mathbf{k}$, initial velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}$, and initial position $\mathbf{r}(0) = 2\mathbf{i} + 3\mathbf{j}$.

Problem 12 (6 points/part)**.** Let $\mathbf{r}(t) = \langle t^2, 2t, \ln(t)\rangle$.

(1) Find the unit tangent vector.
(2) Find the the unit normal vector.
(3) Find the the binormal vector.
(4) Find the the curvature.

Problem 13 (10 points)**.** Let f be the function of two variables given by $f(x,y) = \sqrt{16 - 4x^2 - y^2}$.

(1) Sketch the domain of f. If the domain has a boundary, be sure to make it clear which parts of the boundary are in the domain and which are not. Give reasons.
(2) Sketch the level curve of this function corresponding to the value $\sqrt{12}$.

Problem 14 (20 points)**.** Find the maximum and minimum values of the function $f(x,y) = -\frac{1}{2}x^2 + y^2 - 2y$ on the closed region bounded by the x-axis and the graph of $y = 1 - x^2$.

Problem 15 (20 points)**.** Find the maximum and minimum values of the function $f(x,y) = (x+y)^4 - \cos(x-y)$ on the closed region bounded by the lines $x+y=1$, $x-y=1$, $x+y=-1$, and $x-y=-1$.

Problem 16 (20 points)**.** Find the maximum and minimum values of the function $f(x,y) = (x+y)^2 + (x-2)^2$ on the closed rectangle with vertices $(0,0)$, $(0,2)$, $(3,0)$, and $(3,2)$.

Problem 17 (9 points)**.** Let h be a function such that $h'(t) = \sin(t^3 - t)$. Suppose $f(x,y,z) = x^2 - xyz + xh(y-z)$. Find $D_3 f(x,y,z)$.

Problem 18 (16 points)**.** Let $f(x,y,z) = xy^2z^3 + \sin(z) - x$. Find an equation for the tangent plane to the level surface $f(x,y,z) = -2$ at the point $(2,1,0)$.

Problem 19 (6 points)**.** Find the volume of the parallelepiped determined by the vectors

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \text{and} \quad \mathbf{c} = -\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

Problem 20 (2 points)**.** Is the expression

$$\langle 2, -3, -18\rangle \times \langle -1, 5, -2\rangle \times \langle -7, 12, -27\rangle$$

a scalar, a vector, or not defined?

Problem 21 (10 points)**.** Consider the curve $x(t) = t$, $y(t) = t^2$, $z(t) = t^3$. Find an equation for the normal plane at the point $(1,1,1)$.

Problem 22 (12 points)**.** The helix $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}$ intersects the curve $\mathbf{s}(t) = (1+t)\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(1,0,0)$. Find the angle of intersection of the curves.