MATH 281 (PHILLIPS): MIDTERM 1 PROBLEMS

Problem 1 (1 point)**.** True or false: The behavior of cross products is bizarre and horrible.

Problem 2 (2 points per part)**.** In each of the following cases, decide whether the given expression is a scalar, a vector, or not defined.

(1) $(\mathbf{j} - \mathbf{u}) \cdot (\mathbf{v} + 2\mathbf{w})$, if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$.
(2) $\langle 2,\ -3,\ -7 \rangle \times \langle -1,\ 5,\ -2 \rangle \times \langle -7,\ 3,\ -3 \rangle$
(3) $\big(\langle 2,\ -3,\ -7 \rangle \cdot \langle -1,\ 5,\ -2 \rangle \big) \langle -7,\ 3,\ -3 \rangle$

Problem 3 (10 points)**.** Let g be a real number. Find the distance from the point $(0,\ -1,\ g)$ to the plane given by $x + 2y - 3z = 2$.

Problem 4 (8 points)**.** Let $\mathbf{u}$ and $\mathbf{v}$ be functions whose values are vectors in $\mathbb{R}^3$, and let $c(s) = \mathbf{u}(s) \cdot \mathbf{v}(s)$. Suppose that

$$\mathbf{u}(3) = \langle 2, 0, -4 \rangle, \qquad \mathbf{u}'(3) = \langle -5, 1, 3 \rangle, \qquad \mathbf{v}(3) = \langle 2, -1, 0 \rangle, \qquad \text{and} \qquad \mathbf{v}'(3) = \langle -3, 0, 1 \rangle.$$

Find $c'(3)$.

Problem 5 (6 points)**.** Find all real numbers s such that the vectors $\langle 2,\ -1,\ 3 \rangle$ and $\langle 3,\ 1-s,\ 1 \rangle$ are orthogonal.

Problem 6 (10 points)**.** Find two *unit* vectors which are orthogonal to both $\langle -3,\ 4,\ -2 \rangle$ and $\langle 1,\ 2,\ -1 \rangle$.

Problem 7 (12 points)**.** Consider the surface in $\mathbb{R}^3$ given by

$$\frac{x^2}{4} + (y-2)^2 + \frac{z^2}{16} = 1.$$

Describe and draw its traces in the xz plane and the plane $x = \sqrt{3}$.

Problem 8 (12 points)**.** Find an equation for the plane that contains the line $x(t) = 3t + 2$, $y(t) = -5t$, $z(t) = 4t + 5$ and contains the point $(1, 2, 3)$.

Problem 9 (12 points)**.** Find parametric equations for the tangent line to the curve $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3,\ t^2,\ 2t \right\rangle$ at the point $\left(\frac{1}{3}, 1, 2 \right)$.

Problem 10 (15 points)**.** Find the length of the curve $\mathbf{r}(t) = 4t\mathbf{i} + \frac{8}{3}t^{3/2}\mathbf{j} + t^2\mathbf{k}$ for $1 \le t \le 2$.

Problem 11 (8 points)**.** Find the domain of the vector valued function

$$\mathbf{w}(t) = \left\langle \frac{1}{t+2},\ -\sqrt{t+7},\ \ln(8-t) \right\rangle.$$

Problem 12 (17 extra credit points)**.** *Without* writing anything in terms of coordinates, prove the product rule for the derivative of the cross product of two vector valued functions. You may assume without proof that a differentiable vector valued function is continuous, and that the limit of a cross product is the cross product of the limits.

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Problem 13 (18 extra credit points)**.** Reparametrize the curve $\mathbf{r}(t) = \langle 3\sin(\ln(t)),\, 4\ln(t),\, 3\cos(\ln(t))\rangle$, for $t > 0$, with respect to arc length measured from the point where $t = 1$ in the direction of increasing t.