MATH 281 (PHILLIPS): MIDTERM 1 SAMPLE PROBLEMS

Problem 1 (2 points). Is the expression $\langle 2, 3, -18 \rangle \cdot \langle -1, 5, -2 \rangle$ a scalar, a vector, or not defined?

Problem 2 (2 points). If $u$, $v$, and $w$ are vectors in $\mathbb{R}^3$, is the expression $u \times v \times w$ a scalar, a vector, or not defined?

Problem 3 (2 points). Is the expression $56 \times \langle 2, 3, -18 \rangle$ a scalar, a vector, or not defined?

Problem 4 (2 points). Is the expression $\langle 2, -3 \rangle \times \langle -1, 5 \rangle$ a scalar, a vector, or not defined?

Problem 5 (2 points). Is the expression

$$\langle 2, -3, -18 \rangle \cdot (\langle -1, 5, -2 \rangle \times \langle -7, 12, -27 \rangle)$$

a scalar, a vector, or not defined?

Problem 6 (2 points). Is the expression $56j \times (2i - 3j + 6k)$ a scalar, a vector, or not defined?

Problem 7 (2 points). Is the expression $26j - (2i - 3j + 6k) \cdot (2i - 3j + 6k)$ a scalar, a vector, or not defined?

Problem 8 (4 points). Let $v = \langle 2, 3, -6 \rangle$ and $w = \langle -1, 5, -2 \rangle$. Find the cosine of the angle between $v$ and $w$.

Problem 9 (8 points). Find parametric equations for the line through the point $(0, 1, 0)$, perpendicular to the line $x = 2t$, $y = 1 - t$, $z = 3t$, and parallel to the plane $x + y + z = 217$.

Problem 10 (7 points). Find an equation for the plane that contains the line $x = 3t$, $y = 1 - t$, $z = 2 - t$ and contains the point $(1, 2, 3)$.

Problem 11 (8 points). Find the traces of the surface $z^2 = x^2 + y^2$ in the $xz$ and $yz$ planes, and in the planes $z = k$ for general $k$. Then identify the surface and sketch it.

Problem 12 (14 points). Let $r(t) = \langle t^{-1}, t^2, -t^{-1} \ln(t) \rangle$. Find a parametric equation for the tangent line to this curve at the point $(1, 1, 0)$.

Problem 13 (7 points). Sketch the curve with the vector equation $r(t) = \langle -t^2, 1 + t^7 \rangle$. Indicate with an arrow the direction in which $t$ increases.

Problem 14 (12 points). Find the length of the curve

$$r(t) = \langle t^2, \sin(t) - t \cos(t), \cos(t) + t \sin(t) \rangle$$

for $0 \leq t \leq \pi$.

Problem 15 (4 points). Find an equation of the sphere that passes through the origin and whose center is $(1, -2, 3)$.

Problem 16 (6 points). Find two unit vectors which are orthogonal to both $\langle 1, 1, 1 \rangle$ and $\langle 2, 0, 1 \rangle$.

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Problem 17 (3 points). Let $s$ and $t$ be real numbers. Let $\mathbf{a} = si - 2sj + 3sk$ and $\mathbf{b} = i - tj - 5tk$. Find $\mathbf{a} \cdot \mathbf{b}$.

Problem 18 (6 points). Let $t$ be a real number. Let $\mathbf{a} = i + e^tj + e^{-t}k$ and $\mathbf{b} = 2i - e^tj - e^{-t}k$. Find $\mathbf{a} \times \mathbf{b}$.

Problem 19 (6 points). Find the vector projection of $\langle 12, 1, 2 \rangle$ onto $\langle -1, 4, 8 \rangle$.

Problem 20 (8 points). Find the distance from the point $(1, -2, 4)$ to the plane given by $3x + 2y + 6z = 5$.

Problem 21 (6 points). Find the angle between the planes given by $9x - 3y + 6z = 2$ and $2y = 6x + 4z$.

Problem 22 (8 points). Find all real numbers $t$ such that the vectors $\langle 1, 2 - t, -1 \rangle$ and $\langle -3, -2, 1 - 2t \rangle$ are orthogonal.

Problem 23 (8 points). Find all real numbers $s$ such that the vectors $\langle -12, s, 4 \rangle$ and $\langle -3, -2, -1 \rangle$ are parallel.

Problem 24 (8 points). Find all real numbers $c$ such that

$$\det \begin{pmatrix} -1 & c & 4 \\ 3 & -2 & -1 \\ -3 & 1 & 2 \end{pmatrix} = 0.$$ 

Problem 25 (6 points). Let $t$ be a real number. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = i + j - k$, $\mathbf{b} = i - j + k$, and $\mathbf{c} = -i + j + tk$.

Problem 26 (7 points). Give an example of three distinct lines $L_1$, $L_2$, and $L_3$ in $\mathbb{R}^3$ such that $L_2$ and $L_3$ are both perpendicular to $L_1$, but $L_2$ and $L_3$ are not parallel.

Problem 27 (7 points). Find a vector equation and parametric equations for the line through the point $(-2, 0, 1)$ and parallel to the line $x = 2t$, $y = 1 - t$, $z = 4 + 3t$.

Problem 28 (7 points). Find a vector equation and parametric equations for the line through the points $(6, 1, -3)$ and $(2, 4, 5)$.

Problem 29 (7 points). Find a vector equation and parametric equations for the line through the point $(-2, 1, 0)$ and perpendicular to both $i + j$ and $j + k$.

Problem 30 (7 points). Find an equation for the plane that contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $x + 2y + 4z = 17$.

Problem 31 (8 points). Find an equation for the plane through the points $(0, 1, 2)$, $(2, -3, 8)$, and $(5, 2, 5)$.

Problem 32 (10 points). Describe and sketch the surface $yz = 4$. Include a description of appropriate traces.

Problem 33 (12 points). Let $\mathbf{r}(t) = \arctan(t)i + e^{-2t}j - (\ln(t)/t)k$. Find $\lim_{t \to \infty} \mathbf{r}(t)$.

Problem 34 (12 points). Let $\mathbf{r}(t) = \arccos(t)i + e^{-2t}j - \sin(t)e^t k$. Find $\mathbf{r}'(t)$.

Problem 35 (15 points). Let $\mathbf{r}(t) = (t^{-1}, \sin(\pi t), t^2/(1 + t^6))$. Find $\int_1^2 \mathbf{r}(t) \, dt$. 
Problem 36 (10 points). Sketch the curve with the vector equation \( \mathbf{r}(t) = (t, t, 1 + \cos(t)) \). Indicate with an arrow the direction in which \( t \) increases.

Problem 37 (10 points). Let \( \mathbf{u} \) and \( \mathbf{v} \) be functions whose values are vectors in \( \mathbb{R}^3 \), and let \( f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t) \). Suppose that
\[
\mathbf{u}(4) = \langle 2, 0, 4 \rangle, \quad \mathbf{u}'(4) = \langle 0, 1, 3 \rangle, \quad \mathbf{v}(4) = \langle 2, -1, 0 \rangle, \quad \text{and} \quad \mathbf{v}'(4) = \langle -3, 0, 1 \rangle.
\]
Find \( f'(4) \).

Problem 38 (8 points). Let \( \mathbf{w} \) and \( \mathbf{w}' \) be functions whose values are vectors in \( \mathbb{R}^3 \), and let \( \mathbf{r}(t) = \mathbf{v}(t) \times \mathbf{w}(t) \). Suppose that
\[
\mathbf{v}(3) = \langle 2, 0, 5 \rangle, \quad \mathbf{v}'(3) = \langle 0, 1, -3 \rangle, \quad \mathbf{w}(3) = \langle -2, -1, 0 \rangle, \quad \text{and} \quad \mathbf{w}'(3) = \langle -3, 0, -1 \rangle.
\]
Find \( \mathbf{r}'(3) \).

Problem 39 (12 points). Let \( \mathbf{r}(t) = (\sin(t), \cos(t), t^{3/2}) \). Find the arc length of this curve over the interval \( 2 \leq t \leq 4 \).

Problem 40 (12 points). Let \( \mathbf{r}(t) = \arctan(2t)i + \cos(t^2 + t)j + t^{3/2}k \). Set up, but do not attempt to evaluate, an integral which gives the arc length of this curve over the interval \( 3 \leq t \leq 5 \).