MATH 281 (PHILLIPS): MIDTERM 1 SAMPLE PROBLEMS

Problem 1 (2 points)**.** Is the expression $\langle 2, 3, -18\rangle \cdot \langle -1, 5, -2\rangle$ a scalar, a vector, or not defined?

Problem 2 (2 points)**.** If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, is the expression $\mathbf{u} \times \mathbf{v} \times \mathbf{w}$ a scalar, a vector, or not defined?

Problem 3 (2 points)**.** Is the expression $56 \times \langle 2, 3, -18\rangle$ a scalar, a vector, or not defined?

Problem 4 (2 points)**.** Is the expression $\langle 2, -3\rangle \times \langle -1, 5\rangle$ a scalar, a vector, or not defined?

Problem 5 (2 points)**.** Is the expression

$$\langle 2, -3, -18\rangle \cdot (\langle -1, 5, -2\rangle \times \langle -7, 12, -27\rangle)$$

a scalar, a vector, or not defined?

Problem 6 (2 points)**.** Is the expression $56\mathbf{j} \times (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ a scalar, a vector, or not defined?

Problem 7 (2 points)**.** Is the expression $26\mathbf{j} - (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) \cdot (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ a scalar, a vector, or not defined?

Problem 8 (4 points)**.** Let $\mathbf{v} = \langle 2, 3, -6\rangle$ and $\mathbf{w} = \langle -1, 5, -2\rangle$. Find the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$.

Problem 9 (8 points)**.** Find parametric equations for the line through the point $(0, 1, 0)$, perpendicular to the line $x = 2t$, $y = 1 - t$, $z = 3t$, and parallel to the plane $x + y + z = 217$.

Problem 10 (7 points)**.** Find an equation for the plane that contains the line $x = 3t$, $y = 1-t$, $z = 2 - t$ and contains the point $(1, 2, 3)$.

Problem 11 (8 points)**.** Find the traces of the surface $z^2 = x^2 + y^2$ in the xz and yz planes, and in the planes $z = k$ for general k. Then identify the surface and sketch it.

Problem 12 (14 points)**.** Let $\mathbf{r}(t) = \langle t^{-1},\, t^2,\, -t^{-1}\ln(t)\rangle$. Find a parametric equation for the tangent line to this curve at the point $(1, 1, 0)$.

Problem 13 (7 points)**.** Sketch the curve with the vector equation $\mathbf{r}(t) = \langle -t^2, 1+t^7\rangle$. Indicate with an arrow the direction in which t increases.

Problem 14 (12 points)**.** Find the length of the curve

$$\mathbf{r}(t) = \langle t^2,\, \sin(t) - t\cos(t),\, \cos(t) + t\sin(t)\rangle$$

for $0 \le t \le \pi$.

Problem 15 (4 points)**.** Find an equation of the sphere that passes through the origin and whose center is $(1, -2, 3)$.

Problem 16 (6 points)**.** Find two *unit* vectors which are orthogonal to both $\langle 1, 1, 1\rangle$ and $\langle 2, 0, 1\rangle$.

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Problem 17 (3 points). Let s and t be real numbers. Let $\mathbf{a} = s\mathbf{i} - 2s\mathbf{j} + 3s\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - t\mathbf{j} - 5t\mathbf{k}$. Find $\mathbf{a} \cdot \mathbf{b}$.

Problem 18 (6 points). Let t be a real number. Let $\mathbf{a} = \mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - e^t\mathbf{j} - e^{-t}\mathbf{k}$. Find $\mathbf{a} \times \mathbf{b}$.

Problem 19 (6 points). Find the vector projection of $\langle 12, 1, 2\rangle$ onto $\langle -1, 4, 8\rangle$.

Problem 20 (8 points). Find the distance from the point $(1, -2, 4)$ to the plane given by $3x + 2y + 6z = 5$.

Problem 21 (6 points). Find the angle between the planes given by $9x - 3y + 6z = 2$ and $2y = 6x + 4z$.

Problem 22 (8 points). Find all real numbers t such that the vectors $\langle 1,\ 2 - t,\ -1\rangle$ and $\langle -3,\ -2,\ 1 - 2t\rangle$ are orthogonal.

Problem 23 (8 points). Find all real numbers s such that the vectors $\langle -12,\ s,\ 4\rangle$ and $\langle -3,\ -2,\ -1\rangle$ are parallel.

Problem 24 (8 points). Find all real numbers c such that

$$\det \begin{pmatrix} -1 & c & 4 \\ 3 & -2 & -1 \\ -3 & 1 & 2 \end{pmatrix} = 0.$$

Problem 25 (6 points). Let t be a real number. Find the volume of the parallelepiped determined by the vectors

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \text{and} \quad \mathbf{c} = -\mathbf{i} + \mathbf{j} + t\mathbf{k}.$$

Problem 26 (7 points). Give an example of three distinct lines L_1, L_2, and L_3 in $\mathbf{R}^3$ such that L_2 and L_3 are both perpendicular to L_1, but L_2 and L_3 are not parallel.

Problem 27 (7 points). Find a vector equation and parametric equations for the line through the point $(-2,\ 0,\ 1)$ and parallel to the line $x = 2t$, $y = 1 - t$, $z = 4 + 3t$.

Problem 28 (7 points). Find a vector equation and parametric equations for the line through the points $(6,\ 1,\ -3)$ and $(2, 4, 5)$.

Problem 29 (7 points). Find a vector equation and parametric equations for the line through the point $(-2,\ 1,\ 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.

Problem 30 (7 points). Find an equation for the plane that contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $x + 2y + 4z = 17$.

Problem 31 (8 points). Find an equation for the plane through the points $(0, 1, 2)$, $(2,\ -3,\ 8)$, and $(5, 2, 5)$.

Problem 32 (10 points). Describe and sketch the surface $yz = 4$. Include a description of appropriate traces.

Problem 33 (12 points). Let $\mathbf{r}(t) = \arctan(t)\mathbf{i} + e^{-2t}\mathbf{j} - (\ln(t)/t)\mathbf{k}$. Find $\lim_{t\to\infty} \mathbf{r}(t)$.

Problem 34 (12 points). Let $\mathbf{r}(t) = \arccos(t)\mathbf{i} + e^{-2t}\mathbf{j} - \sin(t)e^t\mathbf{k}$. Find $\mathbf{r}'(t)$.

Problem 35 (15 points). Let $\mathbf{r}(t) = \langle t^{-1},\ \sin(\pi t),\ t^2/(1 + t^6)\rangle$. Find $\int_1^2 \mathbf{r}(t)\, dt$.

Problem 36 (10 points)**.** Sketch the curve with the vector equation $\mathbf{r}(t) = \langle t, t, 1 + \cos(t)\rangle$. Indicate with an arrow the direction in which t increases.

Problem 37 (10 points)**.** Let $\mathbf{u}$ and $\mathbf{v}$ be functions whose values are vectors in $\mathbb{R}^3$, and let $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$. Suppose that

$$\mathbf{u}(4) = \langle 2, 0, 4\rangle, \qquad \mathbf{u}'(4) = \langle 0, 1, 3\rangle, \qquad \mathbf{v}(4) = \langle 2, -1, 0\rangle, \qquad \text{and} \qquad \mathbf{v}'(4) = \langle -3, 0, 1\rangle.$$

Find $f'(4)$.

Problem 38 (8 points)**.** Let $\mathbf{w}$ and $\mathbf{w}$ be functions whose values are vectors in $\mathbb{R}^3$, and let $\mathbf{r}(t) = \mathbf{v}(t) \times \mathbf{w}(t)$. Suppose that

$$\mathbf{v}(3) = \langle 2, 0, 5\rangle, \quad \mathbf{v}'(3) = \langle 0, 1, -3\rangle, \quad \mathbf{w}(3) = \langle -2, -1, 0\rangle, \quad \text{and} \quad \mathbf{w}'(3) = \langle -3, 0, -1\rangle.$$

Find $\mathbf{r}'(3)$.

Problem 39 (12 points)**.** Let $\mathbf{r}(t) = \langle \sin(t), \cos(t), t^{3/2}\rangle$. Find the arc length of this curve over the interval $2 \leq t \leq 4$.

Problem 40 (12 points)**.** Let $\mathbf{r}(t) = \arctan(2t)\mathbf{i} + \cos(t^2 + t)\mathbf{j} + t^{3/2}\mathbf{k}$. Set up, but do not attempt to evaluate, an integral which gives the arc length of this curve over the interval $3 \leq t \leq 5$.