MATH 281 (PHILLIPS): MIDTERM 2 PROBLEM LIST

Problem 1 (1 point)**.** The Squeeze Theorem was invented for the purpose of:
(a) Torturing calculus students.
(b) Torturing functions.
(c) Torturing calculus professors who have to teach it.

Problem 2 (15 points)**.** Find an equation for the normal plane to the parametric curve given by $\mathbf{r}(t) = \langle \cos(t-1),\, 1/t,\, -2t \rangle$ at the point $(1,\, 1,\, -2)$.

Problem 3 (12 points)**.** Let

$$h(x, y, z) = 3x^{65} + xy\sin(y)\cos(z) + \arctan(z^{2020}) + \pi^2.$$

Find $D_2 h(x, y, z)$ (in the book usually called $h_y(x, y, z)$).

Problem 4 (10 points)**.** Find the domain of the function $f(x, y) = \sqrt{x} - \ln(y+1)$. Give reasons.

Problem 5 (12 points)**.** A function f of two variables satisfies

$$f(7, 6) = 11, \qquad f(7, 0) = 2, \qquad f(0, 6) = -3,$$

$$D_1 f(7, 6) = -2, \qquad D_1 f(7, 0) = 8, \qquad D_2 f(7, 6) = 3, \qquad \text{and} \qquad D_2 f(0, 6) = 15.$$

Use the linear approximation (tangent plane approximation) to estimate the number $f(6.95, 6.02)$. (You will not need to use all the information provided.)

Problem 6 (12 points)**.** Let f be a differentiable function of two variables. (In the notation for partial derivatives, call them x and y.) Let $g(s, t) = f(\cos(t) - s,\, 2s^2 + 3t)$. Suppose

$$f(0, 2) = 7, \qquad g(0, 2) = 3, \qquad f_x(0, 2) = -5, \qquad \text{and} \qquad f_y(0, 2) = 2,$$

and

$$f(1, 0) = 3, \qquad g(1, 0) = 7, \qquad f_x(1, 0) = 3, \qquad \text{and} \qquad f_y(1, 0) = -4.$$

Find $g_s(1, 0)$. (You will not need to use all the information provided.)

Restated with partial derivatives labelled by position (ignore this if the version above is clear): Suppose

$$f(0, 2) = 7, \qquad g(0, 2) = 3, \qquad D_1 f(0, 2) = -5, \qquad \text{and} \qquad D_2 f(0, 2) = 2,$$

and

$$f(1, 0) = 3, \qquad g(1, 0) = 7, \qquad D_1 f(1, 0) = 3, \qquad \text{and} \qquad D_2 f(1, 0) = -4.$$

Find $D_1 g(1, 0)$.

Problem 7 (16 points)**.** Let $f(x, y, z) = xy^2z^3 + \sin(z) - x$. Find a unit vector in the direction in which f *decreases* the fastest at the point $(2, 1, 0)$. At what rate does f decrease in this direction?

Problem 8 (10 points)**.** Consider the surface in $\mathbb{R}^3$ given by

$$\frac{(x-3)^2}{4} + y^2 + \frac{z^2}{16} = 1.$$

Describe and draw its trace in the plane $z = \sqrt{12}$. In your graph, be sure to label the axes and put scales on the axes.

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Problem 9 (12 points)**.** Let $\mathbf{u}$ and $\mathbf{v}$ be functions whose values are three dimensional vectors. Suppose that $\mathbf{u}'(t) = -7\mathbf{u}(t)$ and $\mathbf{v}'(t) = 2\mathbf{v}(t) + \mathbf{i}$. Find $\dfrac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t))$. (Your answer might involve $\mathbf{u}(t)$ and $\mathbf{v}(t)$, but may not involve $\mathbf{u}'(t)$ and $\mathbf{v}'(t)$.) Be sure to simplify your answer.

Extra credit problems.

Extra credit. (Do not attempt these problems until you have done and checked your answers to all the ordinary problems on this exam. They will only be counted if you get a grade of B or better on the main part of this exam.)

Problem 10 (15 extra credit points)**.** Define

$$Q(x_1, x_2, x_3, x_4) = x_1^2 + x_2^3 + \arctan(x_3 x_4) + \cos(x_1^2 - 2x_1 x_4) + 2x_4.$$

Find an equation for the tangent hyperplane to the level hypersurface of this function which goes through the point $(-2,\ 3,\ 0,\ -1)$.

All work must be shown, using fully correct notation; no credit for just the answer. In particular, intermediate mistakes will lose many points even if they don't lead to an incorrect final answer.

Problem 11 (15 extra credit points)**.** Find a differentiable function f such that

$$D_1 f(x, y) = x y^2 \cos(xy) + 7 \qquad \text{and} \qquad D_2 f(x, y) = x^2 y \cos(xy) + 2y$$

for all x and y, or explain why no such function exists.

All the credit is for the method; no credit for just writing down the answer.