MIDTERM 2 SAMPLE PROBLEMS

Midterm 2 will cover through the end of Section 14.6 of the book. Emphasis will be on material covered since the first midterm, but it is cumulative, and material from before the first midterm will appear on Midterm 2.

Most problems will be similar to `WeBWorK` problems, written homework problems, problems from the real and sample Midterm 1, and the problems here. Note, though, that the exact form of functions which appear could vary substantially. In particular, a problem requiring computation of the limit or derivative of a single variable function could be modified to use any other single variable function; the methods required to find the limit or derivative could be very different. (This includes computations of partial derivatives, which are really just derivatives of single variable functions.)

Be sure to get the notation right! (This is a frequent source of errors.) The right notation will help you get the mathematics right, and I will complain about incorrect notation.

This handout contains two parts. First is an old exam, from a slightly different course, and which doesn't include anything from Midterm 1. Second is a collection of extra problems. The problems have point values attached, which give a rough idea of the point values problems requiring a similar amount of work will have on the real exam. Note, however, that the real midterm will total 100 points, and the problems here total much more than that.

An old Midterm 2, from a sightly different course and without any review problems.

1. (10 points.) Describe the domain of the function $f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$. Give reasons.

2. (10 points/part.) Let $\mathbf{r}(t) = \langle 8t, 3\sin(2t), 3\cos(2t) \rangle$.

   (a) Find the unit tangent vector.

   (b) Find the unit normal vector.

3. (15 points.) Sketch the graph of the function $f(x, y) = \cos(x)$.

4. (10 points.) Find $\lim_{(x,y)\to(0,0)} 2(x^2 + y^2)\cos\left(\dfrac{7x}{x^2 + y^2}\right)$, giving reasons for your answer, or explain why it does not exist.

5. (15 points.) Let $f(x, t) = x^2 e^{-ct}$, where c is a constant. Find f_{ttxx}.

6. (10 points.) Let F, u, and v be differentiable functions of two variables, and let $W(s, t) = F(u(s, t), v(s, t))$. Suppose that

$$\begin{aligned} u(1,0) &= 2, & u_s(1,0) &= -2, & u_t(1,0) &= 6, \\ v(1,0) &= 3, & v_s(1,0) &= -5, & v_t(1,0) &= 4, \\ F(2,3) &= -7, & F_u(2,3) &= -1, & F_v(2,3) &= 10, \\ F(3,2) &= 7, & F_u(3,2) &= 1, \quad \text{and} & F_v(3,2) &= -10. \end{aligned}$$

Find $W_t(1, 0)$.

7. (10 points.) A function g of two variables satisfies $g(3, 8) = 11$, $g_x(3, 8) = 2$, and $g_y(3, 8) = -6$. Use the linear approximation (tangent plane approximation) to estimate $g(3.1, 7.95)$.

8. (10 points.) Find the maximum rate of change of the function $f(r, s, t) = r^2 s^3 t^4$ at the point $(1, 1, 1)$, and the direction in which it occurs.

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Additional problems.

9. (10 points/part.) Let f be a scalar function which satisfies $f'(t) = \sin(t^3)$, let $\mathbf{u}$ be a vector function which satisfies $\mathbf{u}'(t) = -t^6\mathbf{u}(t)$, and let $\mathbf{v}$ be a vector function which satisfies $\mathbf{v}'(t) = -2\mathbf{v}(t) + \mathbf{i}$.

(a) Find $\frac{d}{dt}(f(t)\mathbf{u}(t))$. (Your answer might involve $f(t)$ and $\mathbf{u}(t)$.) Be sure to simplify your answer.

(b) Find $\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t))$. (Your answer might involve $\mathbf{u}(t)$ and $\mathbf{v}(t)$.) Be sure to simplify your answer.

(c) Find $\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t))$. (Your answer might involve $\mathbf{u}(t)$ and $\mathbf{v}(t)$.) Be sure to simplify your answer.

(d) Find $\frac{d}{dt}(\mathbf{u}(f(t)))$. (Your answer might involve $\mathbf{u}(t)$ and $f(t)$.) Be sure to simplify your answer.

10. (9 points.) Let $\mathbf{v}$ be a vector function which satisfies $\mathbf{v}'(t) = -t\mathbf{v}(t) + 3\mathbf{k}$. Let $\mathbf{r}(t)$ be the vector valued function $\mathbf{r}(t) = \mathbf{v}(t^7)$. Find $\mathbf{r}'(t)$.

11. (8 points.) Let $q(x, y, z) = \sec(2x^3 + 7y^2)z + e^{x^2 z} + \frac{1}{e}$. Find $D_2 q(x, y, z)$.

12. (12 points.) Define a function F by $F(t) = \int_0^t e^{-s^2}\, ds$. Set $f(x, y) = F(7x - y^2)$. Find the first order partial derivatives of f.

13. (15 points.) Find the linear approximation to the function $h(x, y) = \sqrt{x^4 + y^2}$ at the point $(2, 3)$, and use it to estimate the number $\sqrt{(2.05)^4 + (2.9)^2}$.

14. (15 points.) Find the linearization of the function $w(x, y) = \sqrt{6x + e^{4y}}$ at the point $(4, 0)$.

15. (16 points.) Let $f(x, y) = \ln(3x + 7y)$. Find all second order partial derivatives of f.

16. (10 points.) Let f be a differentiable function of three variables. Let x, y, and z be differentiable functions of two variables, and set $w(s, t) = f(x(s, t),\, y(s, t),\, z(s, t))$. Find $w_s(s, t)$ in terms of the functions f, x, y, and z and their partial derivatives.

17. (10 points.) Let f be a differentiable function of two variables. Let $x(s, t) = e^s \cos(t)$ and $y(s, t) = e^s \sin(t)$, and set $u(s, t) = f(x(s, t),\, y(s, t))$. Find $u_t(s, t)$ in terms of s, t, and the function f and its partial derivatives.

18. (10 points.) Let f be a differentiable function of two variables. (In the notation for partial derivatives, call them x and y.) Let $g(r, s) = f(3r - s - 1,\, s^2 - 4r)$. Suppose

$$f(0, 0) = 3, \qquad g(0, 0) = 6, \qquad f_x(0, 0) = 4, \qquad \text{and} \qquad f_y(0, 0) = 8,$$

and

$$f(1, 2) = 6, \qquad g(1, 2) = 3, \qquad f_x(1, 2) = 2, \qquad \text{and} \qquad f_y(1, 2) = 5.$$

Find $g_s(1, 2)$.

19. (15 points.) Draw a contour map of the function $f(x, y) = x^2 + 1 + \sin(y)$, showing level curves for at least three values in the range.

20. (10 points.) Find $\lim_{(x,y)\to(6,3)} xy\cos(x - 2y)$, giving reasons for your answer, or explain why it does not exist.

21. (10 points.) Find $\lim_{(x,y)\to(0,0)} \frac{2}{x^2 + y^4}$, giving reasons for your answer, or explain why it does not exist.

22. (10 points.) Find $\lim_{(x,y)\to(0,0)} \cos\left(\frac{7x}{x^2 + y^2}\right)$, giving reasons for your answer, or explain why it does not exist.

23. (10 points.) Find $\lim_{(x,y)\to(6,3)} \dfrac{x(y-3)}{(x-2)(y^2-2y-3)}$, giving reasons for your answer, or explain why it does not exist.

24. (12 points.) Find the directional derivative of $f(x,y) = x\sin(xy)$ at the point $(2,0)$ in the direction determined by the angle $\pi/3$.

25. Let $f(x,y,z) = \sqrt{x+yz}$.

(a) (8 points.) Find $\mathbf{grad}(f)$.

(b) (4 points.) Evaluate $\mathbf{grad}(f)$ at the point $P = (1,3,1)$.

(c) (6 points.) Find the rate of change of f at the point P in the direction $\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \rangle$.

26. (12 points.) Find an equation for the tangent plane to the surface $x - z = 4\arctan(yz)$ at the point $(1+\pi,\ 1,\ 1)$.

27. (12 points.) Find parametric equations for the normal line to the surface $x - z = 4\arctan(yz)$ at the point $(1+\pi,\ 1,\ 1)$.

28. (12 points.) Explain why there is no function f such that $f_x(x,y) = 3x\sin(y)$ and $f_y(x,y) = 2x^2\cos(y)$.

29. (10 points.) Find an equation for the tangent plane to the surface $z = e^{x^2-y^2}$ at the point $(1,\ -1,\ 1)$.