Black does have \( IR \) for set of real \( \mathbb{R} \), and \( IR \times IR \times IR \) for 3-d space. Abbrev. to \( IR^3 \) is standard notation.

Vector \( IR^3 \) has 3 coordinates ("components"), just like a point. Movement is different here; but think of it in different ways (11 apples vs 3°C).

Used for motion. Draw in \( IR^2 \) (i.e. 2 coords)

Vector now represents a "displacement vector," "from \( (1,1) \) to \( (1,4) \)."


grounded to \( IR \), and \( 4 \cdot 4 \) in written \( \langle 1, 3 \rangle \).

\[ \text{from } (2, 8) \text{ to } (3, 3) \text{ is the same displacement } \langle 1, 3 \rangle \]

Positive vector of \( \langle 1, 3 \rangle \)

Add coord wise: \( \vec{v} = \langle v_1, v_2 \rangle \), \( \vec{w} = \langle w_1, w_2 \rangle \) then \( \vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle \).

Zero vector \( \vec{0} = \langle 0, 0 \rangle \). Still span of nothing; wrote \( \vec{0} \) for vector \( \vec{0} \) of any size for constant \( \vec{0} \) with value \( 0 \)

for point \( \langle 0, 0, \ldots \rangle \) in \( IR^n \).

\[ \vec{v} + \vec{0} = \vec{v} + \vec{v} = \vec{v} \text{ for any vector } \vec{v}. \]

Negative scalars (vectors)

If \( c \) is real \( \mathbb{R} \) and \( \vec{v} = \langle v_1, v_2 \rangle \) a vector, then \( c\vec{v} = \langle cv_1, cv_2 \rangle \) (vector).


Algebraic properties (check that they work): \( \vec{u}, \vec{v}, \vec{w} \) vectors (of same dimension), \( c \) scalar (real number). Then:

\[ \vec{v} + \vec{w} = \vec{w} + \vec{v} \]

\[ (\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w}) \]

\[ \vec{v} + \vec{0} = \vec{v} \]

\[ \vec{0} + \vec{v} = \vec{v} \]

\[ -1 \vec{v} = -\vec{v} \text{ in coords } \langle -v_1, -v_2 \rangle. \]

\[ c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w} \]

\[ (c + d)\vec{v} = c\vec{v} + d\vec{v} \]

\[ c_1 c_2 \vec{v} = c_1 (c_2 \vec{v}) \]

\[ c \vec{v} \]

\[ 1 \times \vec{v} = \vec{v}. \]
Length: \( \|v\| \) (often in book usually \( |v|\)) is the length.

\[ \|v\| = \sqrt{v_1^2 + v_2^2}, \quad \|v_1, v_2, v_3\| = \sqrt{v_1^2 + v_2^2 + v_3^2}, \quad \text{similarly in } \mathbb{R}^n. \]

Property: not written in book: a scalar, then \( \|cv\| = |c| \cdot \|v\| \)

\[ \|v + w\| \leq \|v\| + \|w\|, \quad \text{truly to prove but} \]

\[ \|0\| = 0. \quad \text{geometrically obvious.} \]

\[ v + w \]

\[ v \]

\[ w \]

\[ v \text{ is a unit vector if } \|v\| = 1. \quad \text{Unit vector in same dir. as } v : \frac{1}{\|v\|} \cdot v \]

If \( v = \langle 3, 4 \rangle \), get \( \|v\| = \sqrt{3^2 + 4^2} = 5 \) and unit vector in same direction is \( \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle \).

"standard basic vector"

\[ \mathbb{R}^2: \quad \hat{i} = \langle 1, 0 \rangle, \quad \hat{j} = \langle 0, 1 \rangle. \]

\[ \mathbb{R}^2: \quad \hat{i}, \hat{j} \]

\[ \mathbb{R}^3: \quad \hat{i} = \langle 1, 0, 0 \rangle, \quad \hat{j} = \langle 0, 1, 0 \rangle, \quad \hat{k} = \langle 0, 0, 1 \rangle. \]

\[ \mathbb{R}^3: \quad \hat{i}, \hat{j}, \hat{k} \]

\[ \mathbb{R}^3: \quad \langle 3, 2, -1 \rangle = 3\hat{i} + 2\hat{j} - \hat{k} \text{ in } \mathbb{R}^3. \]

\[ \langle 3, 2, -1 \rangle + \langle 1, 3, -2 \rangle = \langle 4, -6, -3 \rangle. \]

\[ \vec{v} \quad \langle 3\hat{i} + 2\hat{j} - \hat{k} \rangle + \langle -8\hat{i} - 2\hat{k} \rangle = 4\hat{i} - 6\hat{j} - 3\hat{k} \]

\[ \text{If forces } F_1, F_2, \ldots, F_n \text{ act on an object, the total force in } F_1 + F_2 + \ldots + F_n = \vec{F}. \]

Get's works in \( \vec{F} = ma \).

\[ \text{vector} \quad \text{vector} \quad \text{scalar}. \]

\[ \text{If object is not moving, then } F_1 + F_2 + \ldots + F_n = 0. \]
Different version of example p. 844.

\begin{align*}
\text{mass } & 6 \text{ kg} \\
\text{need not be same height.}
\end{align*}

What is the tension in each of the two diving ropes [the forces they exert].
Assume all ropes have no mass. Tensions \( S, T \).

This is located on the planet Yuggoth, where grav acceleration at surface is exactly \( 4 \text{ m/sec}^2 \).

Look at force \( F \) exerted by mass. 2-d problem.

\[ F = -(6) \hat{\mathbf{j}} = -24 \hat{\mathbf{j}} \]

Look at \( \mathbf{T} \):

\[ \mathbf{T} = T_1 \hat{\mathbf{i}} + T_2 \hat{\mathbf{j}} \]

Relation:

\[ T_1 = \| \mathbf{F} \| \cos (0.4) \]
\[ T_2 = \| \mathbf{F} \| \sin (0.4) \]

Therefore

\[ \mathbf{T} = \| \mathbf{F} \| \cos (0.4) \hat{\mathbf{i}} + \| \mathbf{F} \| \sin (0.4) \hat{\mathbf{j}} \]

\[ \text{Similarly: } \mathbf{S} = -\| \mathbf{S} \| \cos (0.6) \hat{\mathbf{i}} + \| \mathbf{S} \| \sin (0.6) \hat{\mathbf{j}} \]

\[ \mathbf{F} + \mathbf{T} + \mathbf{S} = 0 \]

\[ -24 \hat{\mathbf{j}} + \| \mathbf{F} \| \cos (0.4) \hat{\mathbf{i}} + \| \mathbf{F} \| \sin (0.4) \hat{\mathbf{j}} - \| \mathbf{S} \| \cos (0.6) \hat{\mathbf{i}} + \| \mathbf{S} \| \sin (0.6) \hat{\mathbf{j}} = 0 \]

In cards:

\[ \| \mathbf{F} \| \cos (0.4) - \| \mathbf{S} \| \cos (0.6) = 0 \]
\[ -24 + \| \mathbf{F} \| \sin (0.4) + \| \mathbf{S} \| \sin (0.6) = 0 \]

Now solve for \( \| \mathbf{S} \| \), \( \| \mathbf{F} \| \).

Then use (4) to get \( \mathbf{T} \) and (5) to get \( \mathbf{S} \).