Two triangles of triangle spanned by two vectors.

Equation of curves and surfaces, 12.5 has lines and planes and 12.6 has quadratics surfaces (ellipsoids, parabolas, etc.).

Sec 10.1 of book has parametrized curves in $\mathbb{R}^2$.

\begin{align*}
  x(t) &= 2 \cos(t), \\
  y(t) &= 2 \sin(t)
\end{align*}

at $t = 0$ 

Start at $(2,0)$ trace out a circle of radius 2 in counterclockwise direction, going over it repeatedly and come to $(2,0)$ (also for $t < 0$).

\begin{align*}
  x(t) &= t \cos(2\pi t), \\
  y(t) &= t \sin(2\pi t)
\end{align*}

for $t \geq 0$.

Spiral (not well drawn)

For a line in $\mathbb{R}^2$, easiest form $\mathbf{x}$ is as a parametrized curve:

Suppose it is to go through $(-1,2,3)$ in direction $\langle -2,4,-7 \rangle$ (distinguish between point and vector is here very artificial).

Put in not natural problem $r(t) = \langle -1,2,3 \rangle + t \langle -2,4,-7 \rangle = \langle -1-2t, 2+4t, 3-7t \rangle$

Parametrization is $r(t) = \mathbf{x}_0 + t\mathbf{v}$ 

$x(t) = -1-2t, y(t) = 2+4t, z(t) = 3-7t$,
Ex. Line segment from \((-1, 2, 3)\) to \((2, 6, 4)\). What is the direction?
\[
\langle 2, 6, 4 \rangle - \langle -1, 2, 3 \rangle = \langle 3, 4, 1 \rangle.
\]
So \(r(t) = \langle -1, 2, 3 \rangle + t \langle 3, 4, 1 \rangle\) for \(0 \leq t \leq 1\). (parametrized to go from first point to second point.)

There are many different parametrizations of the same line (or curve, ...)

First ex: \(x(t) = -1 + 2t, y(t) = 2 + 4t, z(t) = 3 - 7t\).

Find equation "solution set form." In \(\mathbb{R}^2\), expect a curve from one equation in two variables.

In \(\mathbb{R}^3\), need two equations (generally)

\[
x = -1 - 2t, \quad y = 2 + 4t
\]

Solve for \(t\).

\[
t = \frac{x + 2}{4} = \frac{-1 - 2t}{4}
\]

so \(-x + 1 = -2 - 2t = -z - 7\).

Two equations: \(-x + 1 = -z - 7\).

An immediate consequence:

Components of \(\langle -2, 4, 7 \rangle\), namely \(-2, 4, \text{ and } 7\) are called "direction numbers."

Two lines \(r(t) = r_0 + tv\), \(s(t) = s_0 + tw\) are parallel if direction vectors are parallel.

Same as: \(v\) is a scalar multiple of \(v\) in \(\mathbb{R}^2\): \(v \wedge w = 0\).

In \(\mathbb{R}^3\), non-parallel lines need not intersect. Ex: \(x = t, y = 0, z = 0\) and \(x = 0, y = 0, z = 1\).

Ex: \(r_1(t) = 2 + t, \quad r_1(t) = -3 - 6t, \quad r_1(t) = 2t, \quad r_1(t) = -3 - 6t, \quad r_1(t) = -3 - 6t\).

Directions are \(v_1 = \langle 1, -1, 2 \rangle\) and \(v_2 = \langle -2, -1, 0 \rangle\). Not proportional.

(check it by showing \(v_1 \times v_2 \neq 0\)).

Do they intersect?

\[x_1(s) = -3 - 2s, \quad x_2(s) = -5, \quad z_1(s) = -1 - 3s.\]

Solve: \(x_1(t) = x_2(s)\), etc. as simultaneous equations:

\[
g + t = 3 - 2s \quad \begin{cases} \text{as } x_1(t) = x_2(s), \\
-3 - t = -5 \quad \text{as } y_1(t) = y_2(s), \\
2t = -1 - 3(s + 3) \quad \text{as } z_1(t) = z_2(s).
\end{cases}
\]

Solve: \(2 + t = 3 - 2(4 + 3)\) and \(2t = -1 - 3(t + 3)\).

\[t = \frac{7}{3}, \quad s = \frac{5}{3} \quad \text{so no solution.}\]

Lines do not intersect. Suppose want to solve \(3a + 2b = 7\)
\(2a - 3b = 8\).

If they do intersect, there is no reason to Suppose want to solve \(3a + 2b = 7\)
\(2a - 3b = 8\), one way: solve for \(a\) in first eqn, put in 2nd.
Planes: If want to give parametric equations, how many parameters? Will need two.
Ex: \( \mathbf{r}(s,t) = \mathbf{r}_0 + s\mathbf{v} + t\mathbf{w} \).
Solution set form needs only one equation: \( ax + by + cz + d = 0 \) (linear eqn).
\( \langle a, b, c \rangle \) has a geometric interpretation: \( \sqrt{a^2 + b^2 + c^2} \) eqn says \( \langle a, b, c \rangle \cdot \langle x, y, z \rangle = 0 \).
That is, the pt in plane has direction vectors ortho to \( \langle a, b, c \rangle \). \( \mathbf{n} = \langle a, b, c \rangle \) is called a
"normal vector" to the plane.

More generally: \( \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 \) is an eqn. in the vector variable \( \mathbf{r} \) for a
plane with normal vector \( \mathbf{n} \) and through \( \mathbf{r}_0 \).