Notation:

In scalar mult., scalars always on left, vectors on the right.

If \( c \) is a scalar, \( \mathbf{v} \) is a vector, then write \( cv \) for product (scalar multiplication), never \( vc \).

Also: \( \frac{1}{c} \mathbf{v} \), better \((\frac{1}{c})\mathbf{v}\) or \( \mathbf{v}/c \), never \( \mathbf{v}/c \).

Matrices:

\[
\begin{pmatrix}
1 & 2 & 7 \\
-3 & 4 & 2 \\
-5 & -2 & -1
\end{pmatrix}
\]

is a matrix.

Other notation:

\[
\begin{pmatrix}
2 & 7 \\
-3 & 4 \\
-5 & -2
\end{pmatrix}
\]

also ok.

\( \det \) never means a determinant. Need to write \( \det \left( \begin{pmatrix} 1 & 2 & 7 \\ -3 & 4 & 2 \\ -5 & -2 & -1 \end{pmatrix} \right) \)

or \( \det \left( \begin{pmatrix} 1 & 2 \\ -3 & -2 \end{pmatrix} \right) \).

However, \( \begin{pmatrix} 1 & 2 & 7 \\ -3 & 4 & 2 \\ -5 & -2 & -1 \end{pmatrix} \) is bad notation, but \textcolor{red}{\textbf{always means a determinant}}

\( \text{never just the matrix.} \)

Last time: distance from a point to a plane. Distinct from \( p \) to \( \mathbf{z} \) point not at \( q \), to \( M \).

\( \text{That is, meant } \textbf{M} \text{ in picture.} \)

Need a normal vector, say \( \mathbf{n} \).

Project \( \mathbf{w} \) to \( \mathbf{n} \), but it norm of \( \mathbf{w} \).

\( \text{Call it } M \)
We get: \[ d = \frac{|n \cdot w|}{\|n\|}. \]

The example was plane \( x + 4y - 2z = 6 \),
\( p = (3, 2, -1) \). For \( q \) take \((6, 0, 0)\)
\( n = \langle 1, 4, -2 \rangle \) and \( w = \langle 3, 2, -1 \rangle - \langle 6, 0, 0 \rangle = \langle -3, 2, -1 \rangle. \)
get \( d = \frac{1}{\sqrt{51}} \). This is the distance from the point to the plane.

Ex. (wait, carry me out unless you want me).
Distance between two parallel planes, say \( M \) and \( N \)? Choose some point \( p \) and \( q \) and
then find distance from \( p \) to \( M \) as above.

Ex: Distance between the skew lines, \( L_1 \) and \( L_2 \).
Method is: Find parallel plane \( M_1 \) containing \( L_1 \) and \( M_2 \) containing \( L_2 \), and find
distance between \( M_1 \) and \( M_2 \).

If \( L_1 \) is given by \( r(t) = (3, 2, -1) + t\langle -4, 1, 0 \rangle \)
\( L_2 \)

If \( L_2 \) is given by \( s(t) = (4, 0, 0) + t\langle 0, 1, 2 \rangle \).

A plane parallel to both \( L_1 \) and \( L_2 \) will have a normal vector
orthogonal to direction of both lines. Here we \( n = \langle -4, 1, 0 \rangle \times \langle 0, 1, 2 \rangle. \)
Now use previous method.

12.6: Surface of quadric type in \( \mathbb{R}^3 \)

Parabola:
In \( \mathbb{R}^2 \):

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Hyperbola:
\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

Ellipse:

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

Parabola:
\[ y = 2x^2 \]
Now three kinds of things: ellipses and hyperbolas

Stick with axes parallel to coordinate axes. (x = 1 in \( \mathbb{R}^2 \) has axes not parallel to coord axes).

See book (or any other similar book) for pictures.

(1) \( x^2 + y^2 + z^2 = 1 \): sphere, center (0,0,0), radius 1.

\[ \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \]: Ellipsoid.

In the x plane, it goes from 

\((-2,0,0) \) to \((2,0,0)\)

\((0,3,0) \) to \((0,3,0)\)

\((0,0,-1) \) to \((0,0,1)\).

Consider intersections of the ellipsoid with a plane. Called here a "trace."

Use for now only planes parallel to the coordinate planes. In any of these, the intersections is a ellipse (e.g., \( z = 0 \), \( z = \frac{1}{2} \)), or empty (\( z = 2 \)) or a single point (at \( z = 1 \)).
Net: (2) $x^2 + y^2 - z^2 = 1$

Hyperboloid of one sheet.

Here, the horizontal traces (intersection with horizontal planes) are ellipses.

At $z = 4$, get $x^2 + y^2 - 16 = 1$, so $x^2 + y^2 = 17$.

Vertical trace (could use x-constant or y-constant) for general one get ellipse.

Both kinds of vertical traces give same results.

Take plane $y = 3$. Get $x^2 + 9 - z^2 = 1$ or $x^2 - z^2 = 8$.

Get hyperbola opening "right and left" in the y direction.

Similar for $x = 11$.

(3) $x^2 - y^2 - z^2 = 1$

Repeat above. Do a different one of same kind: $z^2 - x^2 - y^2 = 1$.

Rewrite as $x^2 + y^2 = z^2 - 1$.

No solutions if $-1 < z < 1$. Got

Hyperboloid of two sheets.

Horizontal traces? Here circles, more generally ellipses.

Vertical traces? Hyperbolas facing up and down.
\[ \frac{z}{3} = \frac{x^2}{4} + \frac{y^2}{6} = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{6}}\right)^2 \]

If we consider the plane \( z = 3 \), we have \( \frac{x^2}{2^2} + \frac{y^2}{3^2} = 1 \). This is an ellipse.