M 12 Oct. 2020 Math 281

Notation:

In scalar mult., scalars always on left, vectors on the right.

If c is a scalar, ~~[illegible]~~ and v is a vector, then write cv for product (scalar multiplication), never $\cancel{vc}$.

Also: $\frac{1}{c} \cdot v$, better $\left(\frac{1}{c}\right)v$ or $c^{-1}v$, never $\cancel{\frac{v}{c}}$.

Matrices:

$$\begin{pmatrix} 1 & 2 & 7 \\ -3 & 4 & 2 \\ -5 & -2 & -1 \end{pmatrix}$$ is a matrix

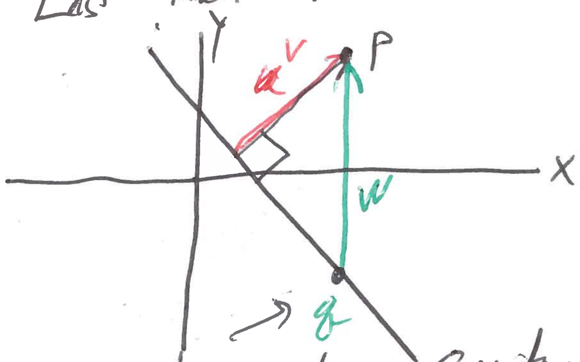
Other notation: $$\begin{bmatrix} 1 & 2 & 7 \\ -3 & 4 & 2 \\ -5 & -2 & -1 \end{bmatrix}$$ also ok.

Neither ever means a determinant. Need to write $\det\begin{pmatrix} 1 & 2 & 7 \\ -3 & 4 & 2 \\ -5 & -2 & -1 \end{pmatrix}$

or $\det\left(\begin{bmatrix} 1 & 2 & \\ & \ddots & \\ & & \ddots \end{bmatrix}\right)$.

However, $$\begin{vmatrix} 1 & 2 & 7 \\ -3 & 4 & 2 \\ -5 & -2 & -1 \end{vmatrix}$$ is bad notation, but always means a determinant, never just the matrix.

Last time: distance from a point to a plane.



z points out at you.

Distance from p to M.

That is, want $\|v\|$ in picture.

Need a normal vector, say n.

Project w to n, take it norm of result.

choose anything.

$\Leftarrow$ intersection of a vertical plane with xy plane. Call it M

We got: $\|v\| = \dfrac{|n \cdot w|}{\|n\|}$.

The example was plane $x+4y-2z=6$.

$p=(3,2,-1)$. For q take $(6,0,0)$

So $n=\langle 1, 4, -2\rangle$ and $w=\langle 3,2,-1\rangle-\langle 6,0,0\rangle=\langle -3, 2, -1\rangle$.

get $\|v\|=\dfrac{8}{\sqrt{21}}$. This is the distance from the point to the plane.

Ex. (won't carry one out unless you want one).

Distance between two parallel planes, say M ~~and~~ and N? Choose some point p on N. Then find distance from p to ~~th~~ M as above.

Any point on N will do.

Ex: Distance between two skew lines, L_1 and L_2.

Method is: Find parallel planes M_1 containing L_1 and M_2 containing L_2, and find distance between M_1 and M_2.

If L_1 is given by $r_1(t)=\langle 3,2,-1\rangle+t\langle -4,1,0\rangle$

L_2 $\quad r_2(t)=\langle 8,0,0\rangle+t\langle 0,1,2\rangle$.

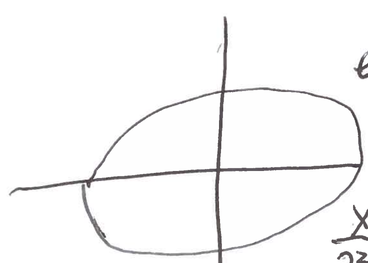
A plane ~~parallel~~ parallel to both L_1 and L_2 will have a normal vector orthogonal to direction of both lines. Here use $n=\langle -4,1,0\rangle\times\langle 0,1,2\rangle$.

Now use previous method.

12.6: surfaces of special types in $\mathbb{R}^3$

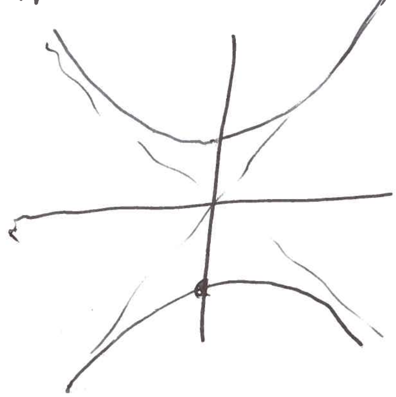
Quadratic:

In $\mathbb{R}^2$:



ellipse

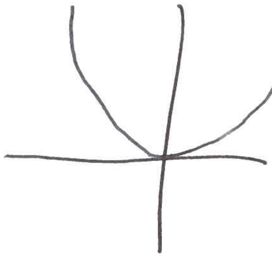
$\dfrac{x^2}{2^2}+y^2=1$



~~hyperp~~ hyperbola

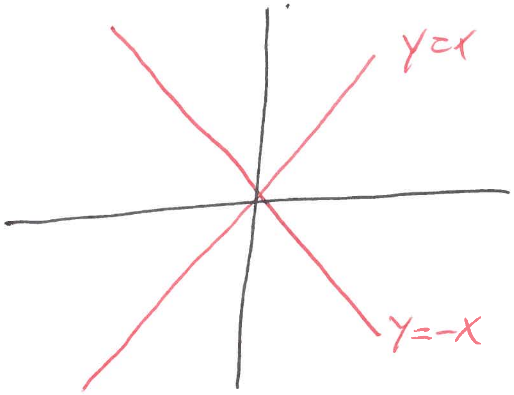
~~$x^2-y^2=1$~~

$y^2-x^2=1$



parabola.

(partially degenerate) $y=2x^2$.



$x^2-y^2=0$. (or $x^2=y^2$)

(Worse: $x^2+y^2=0$ ~~...~~ gives just the point $(0,0)$.
$x^2+y^2=-6$ gives nothing.)

Now <u>three</u> kinds of things like ellipses and hyperbolas
two " " parabolas
one crossing lines.

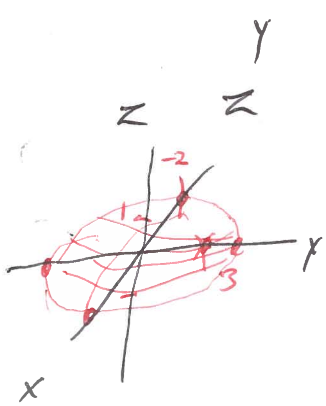
Stick with: axes parallel to coordinate axes. ($xy=1$ in $\mathbb{R}^2$ has axes <u>not</u> parallel to coord axes).
usually center at $(0,0,0)$.

See book (or any other similar book) for pictures.

(1) $x^2+y^2+z^2=1$: sphere, center $(0,0,0)$, radius 1.

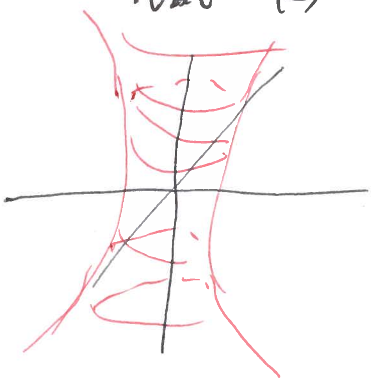
$\frac{x^2}{4}+\frac{y^2}{9}+z^2=1$: <u>ellipsoid</u>.

In the x direction, it goes from ~~(...)~~ $(-2,0,0)$ to $(2,0,0)$
$(0,-3,0)$ to $(0,3,0)$
$(0,0,-1)$ to $(0,0,1)$.



Consider intersection of the ellipsoid with a plane. Called here a "trace."
Use for now only planes parallel ~~...~~ to the coordinate planes. In any of these, the intersection is a ellipse (exs: $z=0$, $z=\frac{1}{2}$), or empty ($z=2$) or a single point (at $z=1$).

Next: (2) $x^2+y^2-z^2=1$



hyperboloid of one sheet:

Here, the horizontal traces (intersections with horizontal planes) are ellipses.

At $z=4$, get $x^2+y^2-16=1$, so $x^2+y^2=17$.

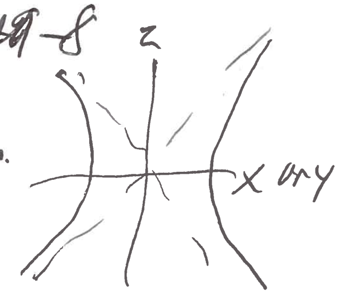
here: circle, but for general ones get ellipse.

Vertical trace (could use x constant or y constant)

Both kinds of vertical traces give same result:

Take plane $y=3$. Get $x^2+9-z^2=1$ or $x^2-z^2=-8$ or $z^2-x^2=8$,

Get hyperbolas opening "right and left" in the y direction.



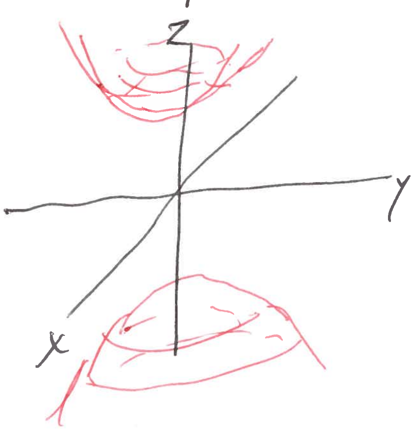
Similar for $x=11$.

(3) $x^2-y^2-z^2=1$

Do a different one of same kind: $z^2-x^2-y^2=1$.

Rewrite as $x^2+y^2=z^2-1$.

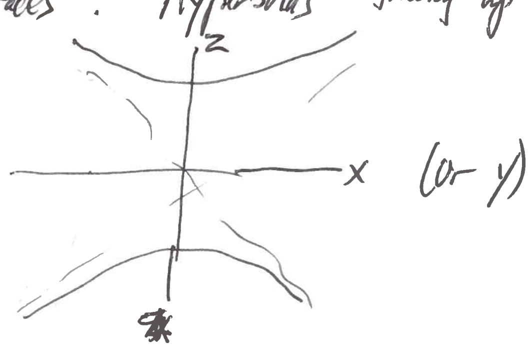
No solutions if $-1<z<1$. Get



hyperboloid of two sheets.

Horizontal traces? Here circles, more generally ellipses

Vertical traces? hyperbolas facing up and down.



$$\frac{z}{3} = \frac{x^2}{4} + \frac{y^2}{16} = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2$$

Or Trace in horizontal plane $z=3$ will be $\frac{x^2}{2^2} + \frac{y^2}{4^2} = 1$, ellipse