Tu 13 Oct. 2020 Math 281

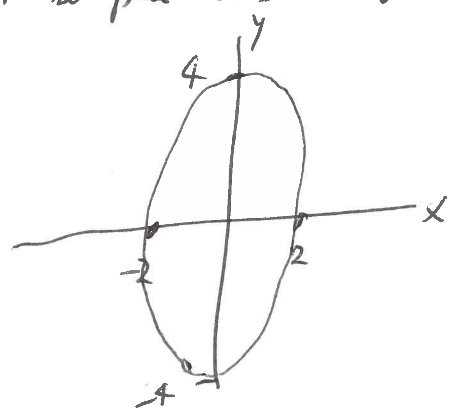
$\frac{z}{3} = \frac{x^2}{2^2} + \frac{y^2}{4^2}$ $\qquad \frac{z}{3} = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2.$

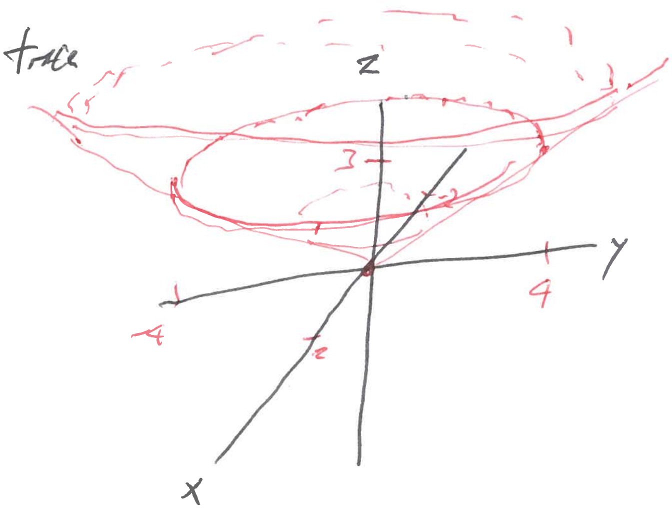
2 and 4 are the half of the two axes of one of traces (an ellipse), namely at $z=3$.

At $z=3$: $\frac{x^2}{2^2} + \frac{y^2}{4^2} = 1$

Therefore the points $(\pm 2, 0, 3)$ and $(0, \pm 4, 3)$ are on the surface.

In the plane $z=3$ we get trace





Hyperbolic paraboloid, basic example is $z = x^2 - y^2$.

Others: $\boxed{\frac{z}{3} = \frac{x^2}{4} - \frac{y^2}{16} = \left(\frac{x}{2}\right)^2 - \left(\frac{y}{4}\right)^2.}$

Picture: see book. (mine is too messy to make sense of)

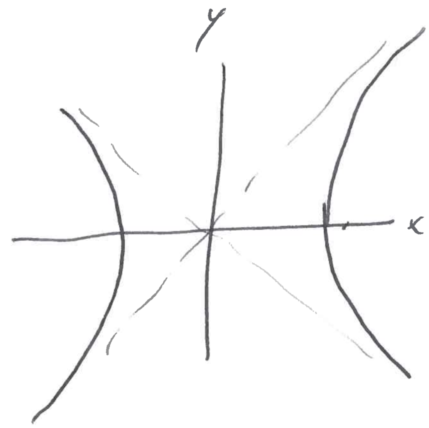
Vertical traces, in planes parallel to xz plane or yz plane will be parabolas, some facing up, some down.

In boxed eqn: If fix $x=3$ get

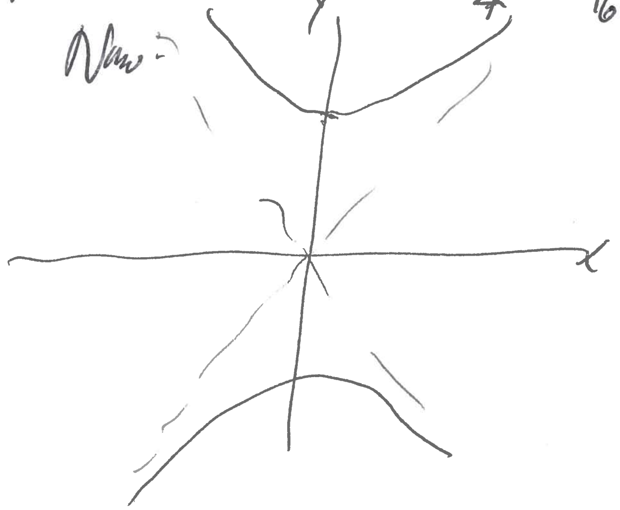
$\frac{z}{2} = \frac{9}{4} - \frac{y^2}{16}$ so $z = \frac{9}{2} - \frac{y^2}{8}$ parabola facing <u>down</u>.

If take $y=4$, get $\frac{z}{3} = \frac{x^2}{4} - 1$, or $z = \frac{3}{4}x^2 - 3$, parabola facing <u>up</u>.

The surface is shaped like a saddle
Horizontal traces will be hyperbolas.
At $z=3$, get $1=\left(\frac{x}{2}\right)^2-\left(\frac{y}{4}\right)^2$



At $z=-6$, get $-2 = \frac{x^2}{4}-\frac{y^2}{16}$
Now:

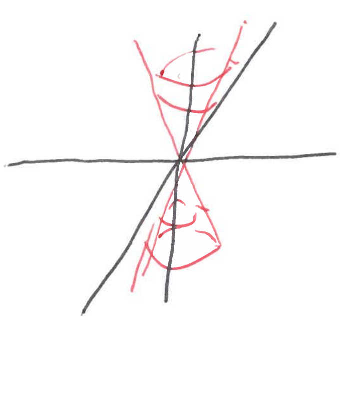


In both pictures, scales not the same on the two axes.

At ~~x~~ $z=0$ get $\frac{x^2}{4}=\frac{y^2}{16}$, two lines which cross: $4x = \pm y$.
$y = \pm 4x$.

$(0,0,0)$ is a ~~sad~~ saddle point
(also: looks like a mountain pass).

Cone (really: a double cone)
Basic eqs is $z^2 = x^2+y^2$



Two cones which meet at the origin.

Example Identify and draw $y^2=x^2+4z^2+2x+5$.

Complete the square in x: $y^2=(x+1)^2+4z^2+4$.

$y^2-(x+1)^2-4z^2=4$

Observe that $-2<y<2$ is not possible: so hyperboloid of two sheets.

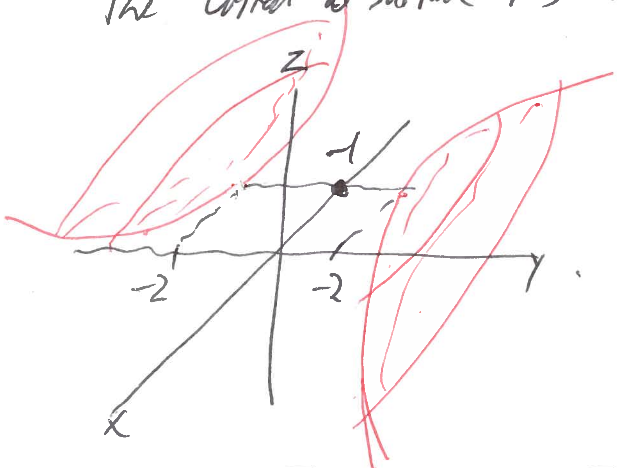
No part of surface between these two planes

The surface is in red. It opens up in directions of positive and negative y-axes.

By mistake, I drew a picture of $y^2-x^2-4z^2=4$.

~~If had $y^2=(x+1)^2$~~

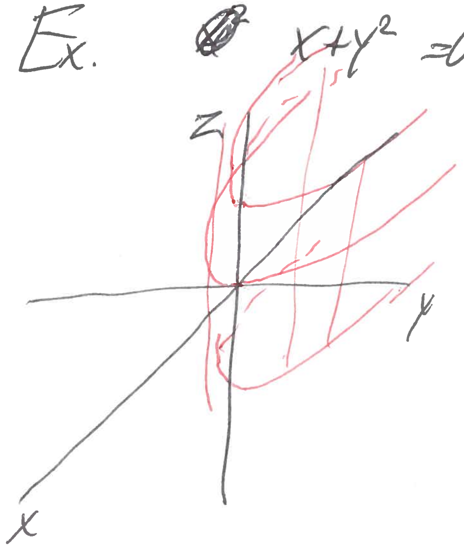
The correct surface has center at $(-1,0,0)$ not at $(0,0,0)$.



"Cylinders" not necessarily quadratic.

Any surface which depends on only two of the three coordinates.

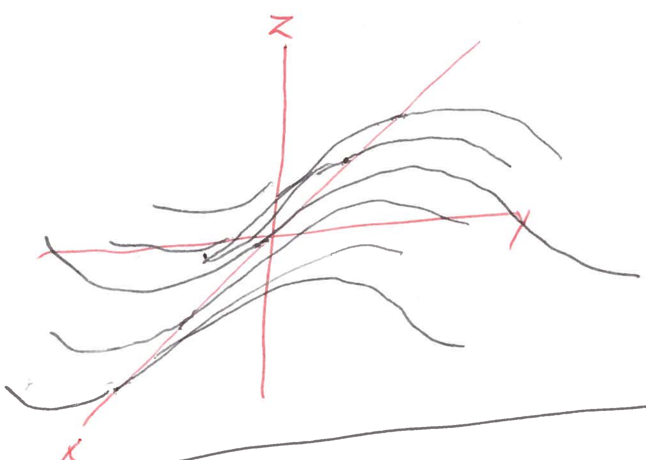
Ex. $x+y^2=0$, or $x=-y^2$



Traces in all horizontal planes are parabolas, always the same: $x=-y^2$

"cylinder" is strange terminology, but is standard (not just here)

Ex $z=\sin(y)$



Now independent of x. Traces in planes "$x=$ constant" are copies of $z=\sin(y)$

Still called a ~~often~~ cylinder. (!)

The conventional kind of cylinder is called here a "right circular cylinder", ~~only get~~ here only get the infinitely long version.

right angle involved.

Sec 13.1 Vector valued functions.

In one variable calculus, have for example a function like

$$f(x)=\frac{1}{\sqrt{4-x^2}}$$ with domain $(-2,2)$, and values in $\mathbb{R}$.

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$f: (-2,2) \longrightarrow \mathbb{R}$. (standard abbreviation; not in book).

$\underbrace{\quad}_{\text{domain}}$ ↑ where the values are

Several ways to generalize:

$r: (-2,2) \longrightarrow \mathbb{R}^2$ or $(-2,2) \longrightarrow \mathbb{R}^3$

Values are ~~now~~ now vectors (or points).

Can consider $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$, a function of three variables

~~$r(t) = \langle \sqrt{1-t^2}, e^{-t}, \frac{1}{\sqrt{t}} \rangle$~~

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Exs $r(t) = \langle \sqrt{1-t^2}, e^{-t}, \frac{1}{\sqrt{t}} \rangle$ This is a function from some subset of $\mathbb{R}$ to $\mathbb{R}^3$, a vector valued function of one variable.

It might describe the position of a particle at time t.

What is its domain? Need $t > 0$ because of the last coordinate. And need $-1 \leq t \leq 1$ because of first coordinate. So domain is $(0, 1]$. (Use both conditions)

Example function of several variables.

$f(x,y,z) = e^{-x} + y^2 + (1+xz)^4(1+\cos(y+z))$.

A function $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ has three coordinate functions, each of which is a function of three variables ← could be the force at position (x,y,z)

density of something at position (x,y,z).

Vector valued functions of one variable are simpler than functions of several variables.

Let's take for a while $r(t) = \langle f(t), g(t), h(t) \rangle$.

Coordinate functions.

Almost everything reduces to the behavior of f, g, h, as in one variable calculus

Ex $\lim_{t \to t_0} r(t) = \left\langle \lim_{t \to t_0} f(t), \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t) \right\rangle$. Need all three limits on the right to exist.

(This is a Theorem).

Ex $\lim_{t \to 0} \left[t^3 i - \frac{e^t - 1}{t} j + \frac{1}{\cos(2t)} k \right]$

$\lim_{t \to 0} t^3 = 0$, $\lim_{t \to 0} \frac{e^t - 1}{t} = 1$, and $\lim_{t \to 0} \frac{1}{\cos(2t)} = 1$.

use L'H rule,
or recognize
defn of deriv. of e^t at 0.

So answer is $-j + k$.

Thm r as above is continuous at t_0 if and only if f, g, h are all continuous at t_0.

The fcn in previous example is continuous on its domain, which is all real numbers except 0, and $\frac{\pi}{4}, \frac{\pi}{2} + \frac{\pi}{4}, \pi + \frac{\pi}{4}, \frac{3\pi}{2} + \frac{\pi}{4}, \dots$

as well as $-\frac{\pi}{4}, -\frac{\pi}{2} - \frac{\pi}{4}, \dots$