MATH 281 (FALL 2020, PHILLIPS): COMMENTS ON AND SOLUTIONS TO HOMEWORK 1

1. Comments on mistakes

On the first written homework, I took off points for only the worst notation mistakes. Please learn from my comments; I will deduct points for more mistakes in the remaining written homeworks *and on exams*.

Follow instructions: Too many people ignored the instructions about assembling two (or more) pages to one file, and about allowed characters in file names (for example, *no spaces in file names*).

A related mistake: a few people didn't follow the definition of the dot product.

Don't write two operation symbols next to each other: Thus,

$$4 + (-x), \qquad \textbf{not} \qquad \cancel{4 + -x},$$

and

$$4 \cdot (-x), \qquad \textbf{not} \qquad \cancel{4 \cdot -x}.$$

The second one is particularly bad, because it is easily read as just $4 - x$.

Never use "$\times$" for multiplication: In mathematics, the symbol "$\times$" should never be used for multiplication in any case. In this course, it is particularly bad, because of confusion with the cross product of vectors in $\mathbb{R}^3$.

Use "=" appropiately: Here is an example (from Math 251) in which sometimes '=" is missing when it is required. Correct:

$$\begin{aligned}
&\lim_{x \to 3} \frac{\sqrt{3x} - 3}{x - 3} \\
&= \lim_{x \to 3} \frac{\left(\sqrt{3x} - 3\right)\left(\sqrt{3x} + 3\right)}{(x - 3)\left(\sqrt{3x} + 3\right)} \\
&= \lim_{x \to 3} \frac{3x - 9}{(x - 3)\left(\sqrt{3x} + 3\right)} \\
&= \lim_{x \to 3} \frac{3}{\sqrt{3x} + 3} \\
&= \frac{1}{2}.
\end{aligned}$$

Date: 4 October 2020.

Do **NOT** write

$$\lim_{x\to 3} \frac{\sqrt{3x}-3}{x-3}$$
$$\lim_{x\to 3} \frac{\left(\sqrt{3x}-3\right)\left(\sqrt{3x}+3\right)}{(x-3)\left(\sqrt{3x}+3\right)}$$
$$\lim_{x\to 3} \frac{3x-9}{(x-3)\left(\sqrt{3x}+3\right)}$$
$$\lim_{x\to 3} \frac{3}{\sqrt{3x}+3}$$
$$\frac{1}{2}.$$

Here is the opposite error:

$$\cancel{x^2+y^2+z^2-4x+6y-2z+10=0}$$
$$\cancel{=x^2+y^2+z^2-4x+6y-2z=-10.}$$

Among other things, this formula claims that $\cancel{0=-10}$, which is obviously false. The correct version is without the "=" at the beginning of the second line:

$$x^2+y^2+z^2-4x+6y-2z+10=0$$
$$x^2+y^2+z^2-4x+6y-2z=-10.$$

Give only one answer: If you give two different answers (as opposed to two different solution methods), one of them *must* be wrong, and you will get at most half credit, probably less.

Work must make mathematical sense: "Show your work" means to show a mathematically correctly written sequence of steps which solves the problem, or mathematically correct reasoning. Your private shorthand (even if you got it from a math teacher as a memory aid) doesn't count.

If $\mathbf{w} = \langle 2, -4, -5\rangle$ and you write

$$\cancel{\|w\| = \sqrt{2^2-4^2-5^2} = \sqrt{4+16+25}},$$

that is *two* mistakes, not just one:

$$\cancel{\|w\| = \sqrt{2^2-4^2-5^2}} \qquad \text{and} \qquad \cancel{\sqrt{2^2-4^2-5^2} = \sqrt{4+16+25}}$$

are both false.

2. Solutions

This homework assignment is due Friday 2 Oct. 2020 at 10:00 pm, to be uploaded as a pdf file (or one of a few other allowed file types) on the University of Oregon Canvas site.

General instructions: show work, and be very careful to use fully correct notation. Incorrect notation will lose credit on exams (grading is based on what you write, not what you meant), and the written homework assignments are your chance to have me tell you whether your notation is correct.

Files turned in must have good enough resolution that I can read them easily.

Apart from the extesion (such as ".pdf"), your file name should contain only numbers, capital and lowercase letters, and underscores. In particular, **no** spaces or parentheses.

Special instructions for this assignment, intended as practice for later. There are two problems. Write your solutions on two separate pages. Then scan or photograph the pages separately, and afterwards combine the images in the correct order into a single file (presumably pdf; **not** Microsoft Word) for submission to Canvas.

Problem 1 (15 points)**.** Define vectors in $\mathbb{R}^3$ by $\mathbf{v} = \langle -1, 3, -2\rangle$ and $\mathbf{w} = \langle 2, -4, -5\rangle$. Find:

(1) $\mathbf{v} - \mathbf{w}$.

Solution. $\mathbf{v} - \mathbf{w} = \langle -1 - 2, 3 - (-4), -2 - (-5)\rangle = \langle -3, 7, -3\rangle$. □

(2) $|\mathbf{w}|$ (also written $\|\mathbf{w}\|$).

Solution. $\|w\| = \sqrt{2^2 + (-4)^2 + (-5)^2} = \sqrt{4 + 16 + 25} = \sqrt{45}$. You can also rewrite the answer as $3\sqrt{5}$, but that is not necessary. □

(3) $(-2)\mathbf{v}$.

Solution. $(-2)\mathbf{v} = \langle (-2)(-1), (-2)(3), (-2)(-2)\rangle = \langle 2, -6, 4\rangle$. □

(4) $\mathbf{v} \cdot \mathbf{w}$ (also written $\langle \mathbf{v}, \mathbf{w}\rangle$).

Solution. $\mathbf{v} \cdot \mathbf{w} = (-2)(-1) + (3)(-4) + (-2)(-5) = -2 - 12 + 10 = -4$. □

(5) $\mathbf{w} \cdot (2\mathbf{w} - \mathbf{v})$ (also written $\langle \mathbf{w}, 2\mathbf{w} - \mathbf{v}\rangle$).

Solution. We have

$$2\mathbf{w} - \mathbf{v} = \langle 2(2) - (-1), 2(-4) - 3, 2(-5) - (-2)\rangle = \langle 5, -11, -8\rangle,$$

so

$$\begin{aligned} \mathbf{w} \cdot (2\mathbf{w} - \mathbf{v}) &= \langle 2, -4, -5\rangle \cdot \langle 5, -11, -8\rangle \\ &= (2)(5) + (-4)(-11) + (-5)(-8) \\ &= 10 + 44 + 40 = 94. \end{aligned}$$

One can also start by writing $\mathbf{w} \cdot (2\mathbf{w} - \mathbf{v}) = 2\mathbf{w} \cdot \mathbf{w} - \mathbf{w} \cdot \mathbf{v}$. This normally takes longer, but you already know that $\mathbf{w} \cdot \mathbf{w} = \left(\sqrt{45}\right)^2 = 45$ and $\mathbf{w} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{w} = -4$. □

In all cases, show at least one intermediate step, and be sure to use enough parentheses.

Problem 2 (10 points)**.** Show that the equation

$$x^2 + y^2 + z^2 - 4x + 6y - 2z + 10 = 0$$

determines a sphere in $\mathbb{R}^3$, and find its center and radius.

Hint: Complete the square. See Chapter 12.1 Example 6 in the book.

Be sure to only write "=" between expressions you are actually claiming are equal, and be sure to use enough parentheses.

Solution. We have

$$\begin{aligned} x^2 + y^2 + z^2 - 4x + 6y - 2z + 10 \\ &= x^2 - 4x + 4 + y^2 + 6y + 9 + z^2 - 2z + 1 + 10 - (4 + 9 + 1) \\ &= (x-2)^2 + (y+3)^2 + (z-1)^2 - 4. \end{aligned}$$

So the equation is

$$(x-2)^2 + (y+3)^2 + (z-1)^2 = 4,$$

which defines a sphere with center $(2, -3, 1)$ and radius $\sqrt{4} = 2$. □