MATH 281 (FALL 2020, PHILLIPS): COMMENTS ON AND SOLUTIONS TO HOMEWORK 5

1. Comments on mistakes

1.1. **Miscellaneous.** Remember to put your name on every page. (Exception: not needed if there are only two pages.) When your homework is printed and graded, the pages are loose. If they are separated, you won't get credit for a page which doesn't have your name on it.

Words are always outside of equations, unless the notation explicitly says otherwise. For example, the phrase

$$\text{“Acceleration at time } 4 = \mathbf{r}''(4)\text{”}$$

is read as "Acceleration at time 4, and $4 = \mathbf{r}''(4)$", which is not what was meant. The right version is:

$$\text{“Acceleration at time 4 is } \mathbf{r}''(4)\text{”.}$$

The right scratchwork version is

$$\text{“(Acceleration at time 4)} = \mathbf{r}''(4)\text{”.}$$

The parentheses are essential. Similarly, writing

$$\text{“Acceleration of } \mathbf{r}(t) = \mathbf{r}''(t)\text{”}$$

includes the incorrect claim $\mathbf{r}(t) = \mathbf{r}''(t)$.

1.2. **Problem 1.** Be careful with graphing ellipses (and hyperbolas). In particular, the graph of

$$\frac{y^2}{16} + z^2 = -\frac{7}{9}$$

is **not** an ellipse, since there are no real numbers y and z which satisfy the equation, and the the graph of

$$\frac{(x-4)^2}{9} + z^2 = \frac{9}{16}$$

is **not** the same as the the graph of

$$\frac{(x-4)^2}{9} + z^2 = 1.$$

Date: 9 November 2020.

1.3. **Problem 2.** Velocity and acceleration are vectors while speed is a scalar. In particular, acceleration is *not* a scalar, and $\|\mathbf{r}''(t)\|$ is not called acceleration. (Possibly there is a name for it, but I don't know one.)

It is simpler to find $\mathbf{r}'(4)$ before calculating $\|\mathbf{r}'(4)\|$ than to use $\|\mathbf{r}'(t)\|$ as the intermediate step. (The judgement on how long an exam is assumes the first method; you are likely to run out of time if on several problems you do things the long way.)

Some people didn't answer the last part. Read the problem!

1.4. **Problem 3.** Since the computation does not work if g is not continuous, a correct justification *must* mention continuity of g.

This has already been pointed out, but many people are *still* making this mistake. It is wrong to write

$$\lim_{(x,y)\to(0,0)} \big(2g(x) - g(x+y+1)\big) = \lim_{(x,y)\to(0,0)} \big(2g(0) - g(0+0+1)\big),$$

because in the second expression the limit has already been evaluated. Writing this looks like a serious misunderstanding of the meaning of a limit, and therefore *always* loses points. It **must** be

$$\lim_{(x,y)\to(0,0)} \big(2g(x) - g(x+y+1)\big) = 2g(0) - g(0+0+1).$$

2. Solutions

Problem 1 (9 points)**.** For the surface

$$\frac{(x-4)^2}{9} + \frac{y^2}{16} + z^2 = 1,$$

find and draw the trace in the xy plane, the trace in the plane $x = 0$, and the trace in the plane $y = \sqrt{7}$. In your graphs, be sure to label the axes and put scales on the axes. Traces are in planes, so draw just the graph in a plane; don't try to locate it in space in your picture.

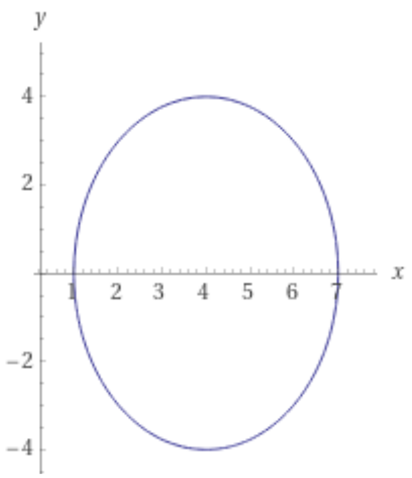
(Before doing this problem, read the list of common mistakes on Problem 1 on Written Homework 4.)

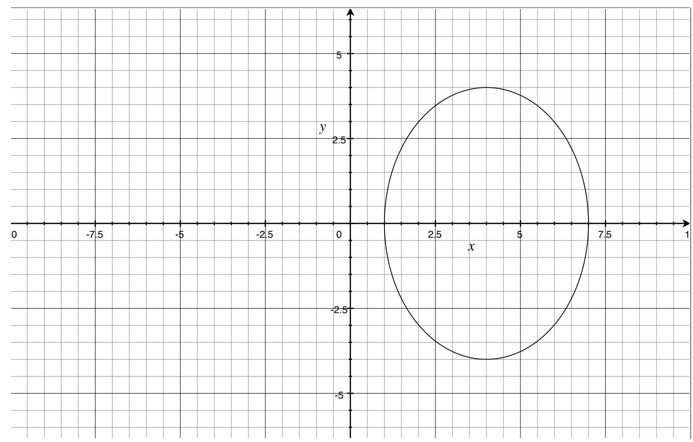
Solution. Although it isn't asked for in the problem, it is helpful to recognize that the surface is an ellipsoid with center at $(4, 0, 0)$. It has "vertices" at $(1, 0, 0)$, $(7, 0, 0)$, $(4, \pm 4, 0)$, and $(4, 0, \pm 1)$.

The trace in the xy plane is gotten by setting $z = 0$:

$$\frac{(x-4)^2}{9} + \frac{y^2}{16} = 1.$$

This is an ellipse with center $(4, 0)$ and vertices at $(1, 0)$, $(7, 0)$, and $(4, \pm 4)$. Here are graphs:





The trace in the plane $x = 0$ is

$$\frac{(0-4)^2}{9} + \frac{y^2}{16} + z^2 = 1,$$

which is

$$\frac{y^2}{16} + z^2 = 1 - \frac{16}{9} = -\frac{7}{9}.$$

This equation has no solutions. The trace is the empty set. (You can just say this and draw nothing; or you can say this and draw axes without a graph. But if you draw axes without a graph, you need to say that the graph is empty, so that I don't think the picture just wasn't drawn.)

The trace in the plane $y = \sqrt{7}$ is

$$\frac{(x-4)^2}{9} + \frac{7}{16} + z^2 = 1,$$

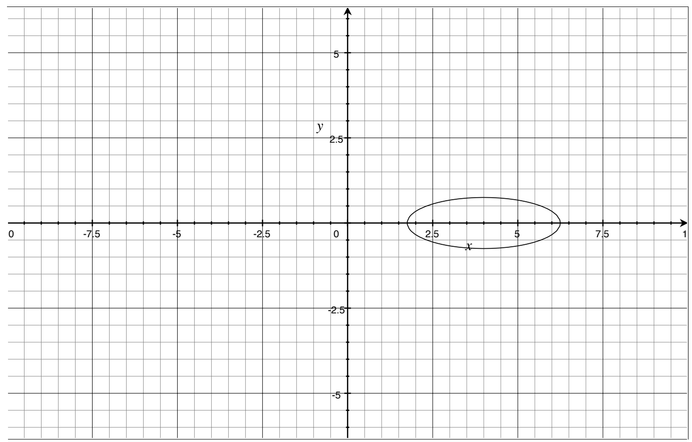
which is

$$\frac{(x-4)^2}{9} + z^2 = 1 - \frac{7}{16} = \frac{9}{16},$$

or

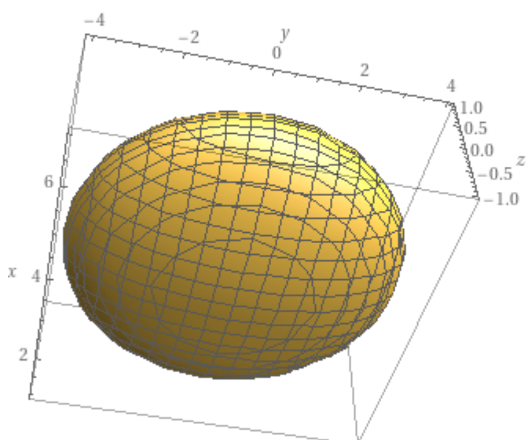
$$\frac{(x-4)^2}{(9/4)^2} + \frac{z^2}{(3/4)^2} = 1.$$

It is an ellipse with center $(4, 0)$ and vertices at $(\frac{7}{4}, 0)$, $(\frac{25}{4}, 0)$, and $(4, \pm\frac{3}{4})$. Here are graphs:



The one on the right shows more, but the vertical axis is mislabelled: it is the z-axis. □

The graph of the surface was *not* asked for, but here it is anyway:



Problem 2 (10 points)**.** The position of a drone flying over part of Corvallis at time t (measured in minutes) is given by

$$\mathbf{r}(t) = \left\langle 6t + 24, \, -th(t), \, h(t) + 5 \right\rangle$$

(measured in meters) for some real valued function h for which you know that

$$h(0) = 11, \qquad h'(0) = 6, \qquad h''(0) = 3, \qquad h'''(0) = 7,$$

$$h(4) = 2, \qquad h'(4) = -3, \qquad h''(4) = 5, \qquad \text{and} \qquad h'''(4) = 1.$$

Find the velocity of the drone at time t (your answer will involve h and its derivatives), the speed of the drone at time 4, and the acceleration of the drone at time 4. Was the drone going up, down, or neither at time 4? Why? (Assume a standard orientation of the coordinate axes.) You will not need to use all the information provided.

Solution. We differentiate coordinatewise, using the product rule on the second coordinate:

$$\mathbf{r}'(t) = \big\langle 6,\, -h(t) - th'(t),\, h'(t) \big\rangle.$$

This is the velocity at time t. So the velocity at time 4 is

$$\begin{aligned}\mathbf{r}'(4) &= \big\langle 6,\, -h(4) - 4h'(4),\, h'(4) \big\rangle \\ &= \big\langle 6,\, -2 - 4(-3),\, -3 \big\rangle = \big\langle 6,\, 10,\, -3 \big\rangle,\end{aligned}$$

and the speed at time 4 is

$$\sqrt{(6)^2 + (10)^2 + (-3)^2} = \sqrt{145},$$

in meters/minute. The acceleration at time t is

$$\mathbf{r}''(t) = \big\langle 0,\, -2h'(t) - th''(t),\, h''(t) \big\rangle.$$

So the acceleration at time 4 is

$$\begin{aligned}\mathbf{r}''(4) &= \big\langle 0,\, -2h'(4) - 4h''(4),\, h''(4) \big\rangle \\ &= \big\langle 0,\, -2(-3) - 4(5),\, 5 \big\rangle = \big\langle 0,\, -14,\, 5 \big\rangle,\end{aligned}$$

in meters/minute^2.

At time 4, the drone was going down because the z coordinate of its velocity was negative. □

Problem 3 (6 points)**.** Let g be a continuous real valued function defined on $\mathbb{R}$ such that $g(0) = 11$ and $g(1) = -3$. Find, with justification,

$$\lim_{(x,y)\to(0,0)} \big(2g(x) - g(x+y+1)\big).$$

Solution. Since g is continuous at 0 and $x \to 0$ as $(x, y) \to (0, 0)$, we have $\lim_{(x,y)\to(0,0)} g(x) = 11$. Since g is continuous at 1 and $x+y+1 \to 1$ as $(x, y) \to (0, 0)$, we have $\lim_{(x,y)\to(0,0)} g(x+y+1) = -3$. The standard properties of limits now give

$$\begin{aligned}&\lim_{(x,y)\to(0,0)} \big(2g(x) - g(x+y+1)\big) \\ &\qquad = 2 \lim_{(x,y)\to(0,0)} g(x) - \lim_{(x,y)\to(0,0)} g(x+y+1) = 2(11) - (-3) = 25.\end{aligned}$$

This completes the solution. □