MATH 281 (PHILLIPS), FALL 2020: WRITTEN HOMEWORK 4 (CORRECTED VERSION)

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Problem 3 has been corrected: the second coordinate of $\mathbf{r}(t)$ is supposed to be $\sqrt{3}t^2$, not $\sqrt{3}t$.

This homework assignment is due Friday 30 Oct. 2020 at 10:00 pm, to be uploaded as a pdf file (or one of a few other allowed file types) on the University of Oregon Canvas site.

General instructions: show work in all problems, and be very careful to use fully correct notation. Incorrect notation will lose credit on exams (grading is based on what you write, not what you meant), and the written homework assignments are your chance to have me tell you whether your notation is correct.

Files turned in must have good enough resolution that I can read them easily.

Apart from the extension (such as "`.pdf`"), your file name should contain only numbers, capital and lowercase letters, and underscores. In particular, **no** spaces or parentheses. You do not need to put any identifying information in the file name; "`HW4.pdf`" is quite sufficient. (The Canvas system adds enough identifying information.)

Write your name on all pages.

Problem 1 (8 points)**.** For the surface

$$\frac{z^2}{16} - \frac{(x-2)^2}{9} - \frac{y^2}{4} = 1,$$

find and draw the trace in the xy plane, the trace in the plane $y = \sqrt{3}$, and the trace in the plane $x = 0$. In your graphs, be sure to label the axes and put scales on the axes. Traces are in planes, so draw just the graph in a plane; don't try to locate it in space in your picture.

Problem 2 (7 points)**.** Let f be a real valued function on $\mathbb{R}$ such that $f(2) = 2$, $f'(2) = -3$, and $f''(2) = 1$. Let $t \mapsto \mathbf{r}(t)$ be the curve in $\mathbb{R}^3$ given by

$$\mathbf{r}(t) = \big\langle f(t),\, tf(t),\, t^2 - 4t \big\rangle.$$

Find $\mathbf{r}'(t)$ (your answer will involve f and its derivatives), $\mathbf{T}(2)$, and the curvature $\kappa(2)$.

Problem 3 (10 points)**.** Let $t \mapsto \mathbf{r}(t)$ be the curve in $\mathbb{R}^3$ given by

$$\mathbf{r}(t) = \big\langle \cos(t^2),\, \sqrt{3}t^2,\, \sin(t^2) \big\rangle,$$

for $t > 0$. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$. Further find equations of the normal and osculating planes at the point where $t = \sqrt{2\pi}$.

Date: 26 October 2020.