MATH 281 (PHILLIPS), FALL 2020: WRITTEN HOMEWORK 6

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This homework assignment is due Friday 13 Nov. 2020 at 10:00 pm, to be uploaded as a pdf file (or one of a few other allowed file types) on the University of Oregon Canvas site.

General instructions: show work in all problems, and be very careful to use fully correct notation. Incorrect notation will lose credit on exams (grading is based on what you write, not what you meant), and the written homework assignments are your chance to have me tell you whether your notation is correct.

Files turned in must have good enough resolution that I can read them easily.

Apart from the extension (such as "`.pdf`"), your file name should contain only numbers, capital and lowercase letters, and underscores. In particular, **no** spaces or parentheses. You do not need to put any identifying information in the file name; "`HW6.pdf`" is quite sufficient. (The Canvas system adds enough identifying information.)

Write your name on all pages.

Problem 1 (9 points)**.** Let h be a function of one real variable such that $h'(t) = e^{-t^3} - t$ for all real numbers t. Let f be the function of three variables given by

$$f(x, y, z) = xyh(z) + \arctan(x^2 + 3z)$$

for all real numbers x, y, and z. Find all first order partial derivatives of f. Your answers may involve the function h.

Problem 2 (4 points)**.** Explain why there is no function g of two real variables such that $g_x(x, y) = 3x^2 + y^3$ and $g_y(x, y) = x^3 + 3xy^2$ for all real numbers x and y.

Hint: consider what the second order partial derivatives would need to be.

Note: showing correct work is important.

Date: 9 November 2020.

Problem 3 (6 points)**.** Let h be a function of three variables such that

$$h_x(x, y) = -2xy + \sin(e^x) \qquad \text{and} \qquad h_y(x, y) = -x^2$$

for all real numbers x and y. Define a function z by $z(t) = h(t^3,\ t\cos(t))$ for all real numbers t. Find $z'(t)$. Remember to simplify your answer!

Problem 4 (6 points)**.** Suppose you have a function f of three real variables which is differentiable at $(3,\ -2,\ -1)$ and such that

$$f(3,\ -2,\ -1) = -8, \quad f(0, 0, 0) = -11,$$

$$f_x(3,\ -2,\ -1) = -2, \quad f_y(3,\ -2,\ -1) = -3, \quad f_z(3,\ -2,\ -1) = -4,$$

$$f_x(3, 0, 0) = -6, \quad f_y(0,\ -2,\ 0) = 3, \quad \text{and} \quad f_z(0,\ 0,\ -1) = 7.$$

Use the linear approximation at $(3,\ -2,\ -1)$ to estimate

$$f(3.02,\ -1.99,\ -1.005),$$

or explain why there is not enough information to do so.

You will not need to use all the information provided.

Be sure to keep the symbols "=" and "≈" straight: use each of them where mathematically appropriate, but don't use one where only the other is correct.