MATH 281 (PHILLIPS), FALL 2020: WRITTEN HOMEWORK 7

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This version has a small addition to the hint for Problem 3.

This homework assignment is due Monday 30 Nov. 2020 at 10:00 pm (note change from regular schedule), to be uploaded as a pdf file (or one of a few other allowed file types) on the University of Oregon Canvas site.

General instructions: This assignment in intended to be done in small groups, but each person must submit their own written version, in their own words, and being sure to understand everything completely.

Show work in all problems, and be very careful to use fully correct notation. Incorrect notation will lose credit on exams (grading is based on what you write, not what you meant), and the written homework assignments are your chance to have me tell you whether your notation is correct.

Files turned in must have good enough resolution that I can read them easily.

Apart from the extension (such as "`.pdf`"), your file name should contain only numbers, capital and lowercase letters, and underscores. In particular, **no** spaces or parentheses. You do not need to put any identifying information in the file name; "`HW7.pdf`" is quite sufficient. (The Canvas system adds enough identifying information.)

Write your name on all pages. On the first page, also write the names of all those people you worked with.

Problem 1 (11 points)**.** Set $g(x, y) = x^2 + 4xy - 2x + 2y^4 - 4y + 1$. Find and classify the critical points of g. Be sure that your work shows that you have found all of them.

Problem 2 (7 points)**.** Let f be a differentiable function of two variables. Let $g(r, s) = f\big(3r^2 s - 1,\, r^2 - s\cos(s)\big)$. Suppose

$$f(1, 0) = 3, \quad g(1, 0) = -7, \quad D_1 f(1, 0) = 4, \quad D_2 f(1, 0) = 8,$$

$$f(-1, 1) = -7, \quad g(-1, 1) = 3, \quad D_1 f(-1, 1) = 2, \quad D_2 f(-1, 1) = -5,$$

and

$$f(3,\, -1) = 11, \quad g(3,\, -1) = -2, \quad D_1 f(3,\, -1) = -9, \quad D_2 f(3,\, -1) = 7.$$

Find $D_2 g(1, 0)$.

Date: 25 November 2020.

Problem 3 (7 points)**.** For $(x, y) \neq (0, 0)$, define

$$h_1(x, y) = -\frac{y}{x^2 + y^2} \qquad \text{and} \qquad h_2(x, y) = \frac{x}{x^2 + y^2}.$$

Show that there is no function f defined for all points (x, y) in $\mathbb{R}^2$ with $(x, y) \neq (0, 0)$ whose first and second order partial derivatives all exist and are continuous on its domain and such that $D_1(x, y)f = h_1(x, y)$ and $D_2(x, y)f = h_2(x, y)$ for all points (x, y) in its domain. Explain why this does not violate Clairaut's Theorem, on page 959 of the book.

Hint. Suppose such a function f exists. Look at what the derivative of the function $q(t) = f(\cos(t), \sin(t))$ would have to be. Use this information to compare $q(2\pi)$ with $q(0)$, and find something wrong with the result you get.