End of yesterday: Somebody commute to work was $11.2 \pm 0.1$ miles. The return trip is $11.5 \pm 0.2$.

(Different, are my streets are involved, and free way ramps are different lengths in different directions.)

The total trip will be $11.2 + 11.5 = \sqrt{(0.1)^2 + (0.2)^2}$ miles.

Assuming $\sigma$ the error is independent of a construction project causes a

0.224

trips, errors are not independent.

If smoothing is $26 \pm 0.132$ lites, then if multiply by 1000, get

$2600 \pm 132$ (multiply both by 1000; in the ex, needed to measure

volume in $\text{ml}$ instead of liters.

Ex. Flip a coin (possibly biased). Let $p$ be the proportion of tails "in the long term"

("the probability that a single flip is tails")

Flip $n$ times, get a "sample proportion" $\hat{p} = \frac{\text{# of tails}}{n}$.

Standard error: $\text{SE} = \sqrt{\frac{n(1-p)}{n}}$.

The prob. of heads is $1-p$. Equivalent to work with this.

Ex. special case. Suppose coin is fair. For every flip, observation is 1 if got tails

(\( \hat{p} = \frac{1}{2} \))

0 if got heads. Error $= 1-\hat{p}$. if got tails, $0 - \hat{p} = -\hat{p}$ if got heads. For a fair coin,

$1-\hat{p} = \frac{1}{2}$ and $-\hat{p} = -\frac{1}{2}$.

RMS (root mean square) is

$$\sqrt{\frac{(\frac{1}{2})^2 + \ldots + (\frac{1}{2})^2}{n}} = \sqrt{\frac{n(\frac{1}{2})^2}{n}} = \frac{1}{2} = \sqrt{\frac{1}{2}(\frac{1}{2})}$$

Then $\frac{1}{2}$ of these, all equal $\frac{1}{2}$.

Thus the RMS error for one flip. By Pythagorean theorem, RMS error for

sum of $n$ flips is $\sqrt{(\frac{1}{2})^2 + \ldots + (\frac{1}{2})^2} = \sqrt{\frac{n}{2} + \frac{n}{2}} = \sqrt{n \cdot \frac{1}{2}}$

To get the proportion, divide by $n$. Get $\left(\frac{1}{2}\right) \frac{1}{n}$, which is equal to $\sqrt{n \cdot \frac{1}{2}}$.

This is a proof of a very special case.

Heuristics for general case, $\text{SE}$ std error for one flip. This is supposed to be, internally, least as $N \rightarrow \infty$ if RMS error for $N$ flips, though can with $N$ many

tails (Should be expected value).
In our $N$ trials, approx $N\pi$ are tails, and approx. $N(1-\pi)$ are heads.

For tails get error $1-\pi$, for heads get $\pi$.

RMS error is: 

$$\sqrt{\frac{\text{[add up count $N\pi$ terms]} + \text{sum of abs $N(1-\pi)$ terms]}{N}}$$

$$\approx \sqrt{\frac{N\pi(1-\pi) + N(1-\pi)\pi}{N}} = \sqrt{N \pi (1-\pi) (1-\pi) + N\pi (1-\pi) \pi}$$

$$= \sqrt{\frac{N(1-\pi)(1-\pi + \pi)}{N}} = \sqrt{\pi(1-\pi)}.$$  
This is the std err for one flip.

Std err for $n$ flips is $\sqrt{\pi(1-\pi)/n}$.

This is the same kind of limit as $N \to \infty$, of $N$ repetitions of:

flip 1, can $n$ trials and look at error in the proportion.

Ex $n = 3$. We do the following many times: flip coin 3 times, get a sample proportion $p = \frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{3}$, and look at error $p - \pi$. Take RMS of $N$ errors of the kind, and let $N \to \infty$. Here $N$ cancels out but $3$ (or general $n$) does not cancel out.

In ex 1 let $1$ to be the number of tails. So error is $t_n - 3\pi$. Std err from the

$$\sqrt{n \pi (1-\pi)}$$ for one flip, is

$$\sqrt{(SE_1)^2 + (SE_2)^2 + (SE_3)^2} = \sqrt{(SE_1)^2} + \text{[two more terms]}$$

$x_1$ after 1st flip, etc. with std err $SE_1$, etc

$$= \sqrt{\frac{3\pi(1-\pi)}{3}} = \sqrt{\pi(1-\pi)} = \sqrt{\pi(1-\pi)/3}.$$  

$p = \frac{1}{3}$, so SE for $p$ is

$$\frac{1}{\sqrt{3}} \sqrt{\pi(1-\pi)} = \frac{1}{\sqrt{3}} \sqrt{\pi(1-\pi)} = \frac{1}{\sqrt{3}} \sqrt{\pi(1-\pi)}.$$  

Lemma: if $p$ is in $[0,1]$, then $\sqrt{\pi(1-\pi)} \leq \frac{1}{2\sqrt{n}}$.

(A hint: we calculate to maximize $f(p) = \pi(1-\pi)$, and see max when $p = \frac{1}{2}$, at $\pi = \frac{1}{2}$.

Ex On Monday, a survey in Megalopolis gave 367 "yes" in a sample size of 96.

Assuming really a SRS ("simple random sample"), what is std error?

We did know $n$, but at $SE \approx \frac{1}{2\sqrt{n}} = \frac{1}{2\sqrt{96}} = 0.016$.

(The true error it $\leq SE$ with confidence about 68%).

For reported polls, usually $

Corresp. number here is about 328
1.9 As part of a 1990 study on the causes of asthma, the parents of 939 seven-year-old children in five German cities were interviewed. Of these children, 57 had had a doctor’s diagnosis of asthma at some time in their lives. Find the statistic $p$ and estimate the standard error of this statistic.

Let’s pretend this was a SRS.

$p = \frac{57}{939}$

Sample proportion \( \hat{p} \) is a statistic.

Estimate: $SE \leq \frac{1}{2 \sqrt{939}} \approx 0.016$ (or 1.6%). (Vey difficult)

Suppose we want $SE \leq 0.01$. How big a sample is needed? (Do it for next time.)