A coin is flipped 10,000 times. It comes up tails ≤ 5132 times. Can we say with 95% confidence that it is not fair? 99.7% confidence?

How to decide: Assume the coin is fair, and see whether the result is unlikely. Assume $p = \frac{1}{2}$. Use $\bar{p}$ (population proportion) to estimate std error. [Note the same as if using p to estimate $\pi$.]

\[
\text{Std error } = \sqrt{\frac{\pi (1-\pi)}{n}} = \frac{1}{\sqrt{200}} = 0.045
\]

So 95% of the time, 10,000 flips should give $\frac{1}{2} - 2(0.045) \leq p \leq \frac{1}{2} + 2(0.045)$

\[
0.49 \leq p \leq 0.51
\]

Actual $\bar{p} = 0.5132 \Rightarrow$ with 95% confidence, coin is unfair.

\[
\frac{5132}{10,000} (0.5132 \text{ is not in } [0.49, 0.51])
\]

For 99.7% confidence, use 3 SE, and do not have 99.7% confidence that coin is unfair.

Set $S$ of outcomes

$Pr(S) \rightarrow 0.17$ tells you the probability of each outcome (in S).

Define An event is a subset of $S$.

Ex: Given flipping a coin once. $S = \{H, T\}$. What are the events?

They are: $\emptyset$, $\{H\}$, $\{T\}$, $\{H, T\}$ (empty set)

Flip coin twice: $S = \{HH, HT, TH, TT\}$. Each: first head is tails is $\{TH, TT\}$

Both heads the same: $\{HH, TT\}$. There are 16 possible events. 4 outcomes: at most one head.

let $E$ be an event. Then $Pr(E) = \sum Pr(S)$.

If coin is fair, prob of first two tails $\frac{2}{4} = \frac{1}{2}$, prob of at most one $H$ is $\frac{3}{4}$.

In first ex, $\{HH, TT\}$ is the set of all outcomes.

$Pr(\{HH, TT\}) = 1$.

Cautions: using HT for the seq. of two flips, first $H$, second $T$.

$\emptyset$ is an event for just one flip.

Ex: Roll a die.

Set outcomes is $\{1, 2, 3, 4, 5, 6\}$.

Thus is an event: $\{2, 4, 6\}$. (The number rolled is even)

If by die is fair, then $Pr(\{2, 4, 6\}) = \frac{\text{card}(\{2, 4, 6\})}{\text{card}(\{1, 2, 3, 4, 5, 6\})} = \frac{3}{6} = \frac{1}{2}$.

\[
\text{And, event: anything but a 4. Take } E = \{1, 2, 3, 5, 6\}. \text{ If fair, } Pr(E) = \frac{5}{6}.
\]
Roll two fair dice. Outcomes are ordered pairs \((1,1), (1,2), \ldots\)...

\[ P(6^2) = \frac{36}{6^2} = \frac{1}{6}. \]

Let \(E\) be the event: total is 2. Then \(E = \{(1,1)\} \) and \(\Pr(E) = \frac{1}{36}. \)

\[ F \]

\[ \text{Then } F = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}. \]

\[ \text{So } \Pr(F) = \frac{6}{36} = \frac{1}{6}. \]

Problem for next time: Prob of total at least 11.

Ex: A biased coin gives tails \(\frac{2}{3}\) of times when flipped. Toss it twice.

\[ \text{What is } \Pr(\text{T}T)? \]

\[ \Pr(\text{T}T) = \left(\frac{2}{3}\right)^2 = \frac{4}{9}. \]

\[ \Pr(\text{T}T, \text{TT}) = \frac{4}{9} + \frac{1}{9} = \frac{5}{9}. \]

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Suppose have two cards \(E, F\). How do find prob of \(\text{"E or F"}\)?

Ex: 1 die, \(E\): roll 1 or 2, \(F\): roll divisible by 3

\[ E = \{2,4,6\}, F = \{3,6\}. \]

Symbol: \(E \cup F\) \(\text{L U is not U; see page 93 of book}\).\n
\[ E \cup F = \{2,3,4,6\}. \]

\[ \text{Rule: } \Pr(E) + \Pr(F) - \Pr(E \cap F) = \Pr(E \cup F). \]

Note: If \(E, F\) are disjoint \((E \cap F = \emptyset)\), then \(\Pr(E \cap F) = 0\) so \(\Pr(E \cup F) = \Pr(E) + \Pr(F)\)

Ex with one fair die: \(E = \{1,3,5\}, F = \{6\}\). Then \(\Pr(E) = \frac{3}{6}, \Pr(F) = \frac{1}{6}\) and \(\Pr(E \cup F) = \Pr(E) + \Pr(F) = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}\)

Ex: Card at random from a standard deck (no jokers). (There are 52 cards, 13 of

What is prob that it is a king or a diamond?

\[ \frac{16}{52} = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \]

\[ \text{Simplify to } \frac{4}{13}. \]

What is it with a std deck but with an extra OK added to it?