SOLUTION TO THE QUESTION FOR MATH 343 FOR THE LECTURE OF 7 APRIL

Problem 1. A coin is flipped 10,000 times. It comes up tails 5,132 times. Can we say with 95% confidence that the coin is biased? Can we say with 99.7% confidence that the coin is biased?

Solution. Let π be the true long run proportion of tails when this coin is flipped. The standard error for this sample is

$$\mathrm{SE} = \sqrt{\frac{\pi(1-\pi)}{10{,}000}}.$$

Here the sample proportion is $p = 0.5132$. If the coin is fair, then $\pi = \frac{1}{2}$, the actual error is $p - \pi = 0.0132$, and the standard error is

$$\mathrm{SE} = \sqrt{\frac{\frac{1}{2}(1-\frac{1}{2})}{10{,}000}} = \frac{1}{2\sqrt{10{,}000}} = 0.005.$$

According to the rule of thumb for confidence, in the long run about 95% of sequences of 10,000 flips will give an actual error satisfying

$$|p - \pi| \leq 2 \cdot \mathrm{SE} = 0.01.$$

Our observed $|p-\pi|$ is bigger, so we have 95% confidence that the coin is not fair. Put differently, we observed something which, if the coin were fair, would occur with probability less than 5%.

According to the rule of thumb for confidence, in the long run about 99.7% of sequences of 10,000 flips will give an actual error satisfying

$$|p - \pi| \leq 3 \cdot \mathrm{SE} = 0.015.$$

Our observed $|p-\pi|$ is smaller, so we do not have 99.7% confidence that the coin is not fair. Put differently, we are asking for an observation which would occur with probability less than 0.3% if the coin were unfair, and that didn't happen. $\square$

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