A simple fair die is rolled. \( E \): result is even. \( F \): result is 4 or more. What is \( P(E|F) \)?

\[ P(E|F) = \frac{P(E \cap F)}{P(F)} \]

\( E = \{ 2, 4, 6 \} \) \( F = \{ 4, 5, 6 \} \) \( \Omega \) \( \{ 1, 2, 3, 4, 5, 6 \} \)

\[ P(E) = \frac{3}{6} = \frac{1}{2}, \quad P(F) = \frac{1}{2}, \quad P(E \cap F) = \frac{1}{2} \]

\[ \frac{3}{1} = \frac{1}{2} \]

\[ \frac{1}{2} \]

Ex. (similar to p. 66 if both).

Draw two cards at random without replacement. What is prob both are queens?

\( Q_1 \): first \( u \) \( Q_2 \): second

\[ \frac{2}{13} \left( \frac{12}{51} \right) = P(\text{both queens}) \]

\( Q_1 \): first \( \frac{1}{13} \)

\( Q_2 \): second \( \frac{12}{51} \)

\[ \frac{9}{65} \]

Second branches: conditional prob.

\[ \frac{12}{51} \]

\( Q_1 \): first \( \frac{12}{51} \)

\( Q_2 \): second \( \frac{9}{65} \)

\[ \frac{9}{65} \]

Find this directly

\[ P(Q_2 | Q_1) = \frac{3}{51} \]

\( Q_1 \): first \( \frac{3}{51} \)

\( Q_2 \): second \( \frac{51}{65} \)

\[ \frac{9}{65} \]

\( Q_1 \): first \( \frac{4}{51} \)

\( Q_2 \): second \( \frac{48}{51} \)

\[ \frac{48}{51} \]

The events here are disjoint, so can add their probabilities.

What is prob both cards are queens? 
\( P(Q_1 \cap Q_2) = \left( \frac{3}{13} \right) \left( \frac{4}{51} \right) = \frac{1}{13 \cdot 17} \)

What is prob that get exactly one \( Q \)?

\[ P(Q \cap Q') + P(Q' \cap Q) = \left( \frac{1}{13} \right) \left( \frac{50}{51} \right) + \left( \frac{12}{13} \right) \left( \frac{4}{51} \right) = \frac{92}{729} \]

For next time: Use this method on:

Ex. A box contains 3 green marbles and three purple marbles. Draw two at random without replacement. What is prob that both have same color? Different colors?
Monty Hall problem 3 doors. At random: car behind one, goats behind other two. Contestant chooses door at random, Host picks, out of those other doors with goats, one at random and opens it. The contestant is then allowed to change choice to the other closed door. Shall the contestant do that?

Let $A_i$ be the event: car behind door $i$. $D_i$: Contestant chooses door $i$.

$H_1$: Event: Monty Hall opens door 1. $H_2$, $H_3$.

What to know: (New points): $E_1$: event that there is a car behind one of the doors the contestant chose. $E_2$: Car behind other closed door. What $\Pr(E_1)$, $\Pr(E_2)$

\[
\begin{align*}
\begin{array}{c|cccc}
A_i & 1 & 2 & 3 & 1 \\
D_i & 2 & D_2 & D_3 & 3 \\
H_1 & 0 & \frac{1}{8} & \frac{1}{8} & 0 \\
H_2 & 0 & \frac{1}{9} & 0 & \frac{1}{9} \\
H_3 & 0 & 0 & \frac{1}{9} & 0 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
&\text{Given $A_2$ or $A_3$, get some results. (Check!)}
&\sum \Pr(E_i) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}.
&\Pr(E_2) = \frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{2}{3}.
\end{align*}
\]

Note: $D_i$ while space $S$ is the disjoint union of $A_1$, $A_2$, $A_3$. Therefore

\[
\Pr(E_1) = \Pr(E_1|A_1) \Pr(A_1) + \Pr(E_1|A_2) \Pr(A_2) + \Pr(E_1|A_3) \Pr(A_3).
\]
For people in age range 50-69, prob of having colon cancer is \(0.001286\) (approx).

Test for it using a colonoscopy. Let \(T\) be the event: positive test result.

We are told (by experiment with test)

\[
\Pr(T|D) = 0.95 \quad (95\% \text{ true positives})
\]
\[
\Pr(T'|D') = 0.86 \quad (86\% \text{ true negatives})
\]

1.4% false positives among tests done on those with disease.

What is \(\Pr(D|T)\)?

(Given a positive test result, what is the prob of nevertheless not having disease?"

\[
\begin{align*}
\Pr(D|T) &= \frac{\Pr(T|D) \cdot \Pr(D)}{\Pr(T)} \\
&= \frac{0.95 \cdot 0.001286}{0.1294996} \\
&= 0.010693 \\
&= 0.010693 \\
&= 0.010693 \\
&= 0.010693 \\
&= 0.010693
\end{align*}
\]

It shall be

\[
\frac{0.010693}{0.1294996} = 0.080122
\]

About 80.1%.

99% of positive test results are in people who don't have disease.

Added afterwards:

What happened? The proportion of people with the disease is so small as to be overwhelmed by the proportion of those who test positive but don't have the disease, even with a test that produces few false positives.

Compare extreme case: If nobody has the disease, all people with positive test results nevertheless don't have the disease. There will be some unless the test never gives false positives, which is not realistic.

What happens to the false positives? Usually followups with other tests, probably more complicated (done on all people with positive tests). If the followup tests have risks, it is possible, depending on the incidence of the disease, risks of followup tests, etc., that more people die of complications of the followup tests than the number of people who have the disease.