Problem 1. Roll one fair standard die. Let $E$ be the event that the roll is even, and let $F$ be the event that the roll is four or more. Find $\Pr(E|F)$. How does it compare with $\Pr(E)$? Are $E$ and $F$ independent?

Solution. We have

$E = \{2, 4, 6\}$, 
$F = \{4, 5, 6\}$, 
and 
$E \cap F = \{4, 6\}$.

So

$\Pr(E) = \frac{3}{6} = \frac{1}{2}$, 
$\Pr(F) = \frac{3}{6} = \frac{1}{2}$, 
and 
$\Pr(E \cap F) = \frac{2}{6} = \frac{1}{3}$.

Therefore

$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$.

This answer is different from $\Pr(E) = \frac{1}{2}$ (it is larger), so $E$ and $F$ are not independent. □