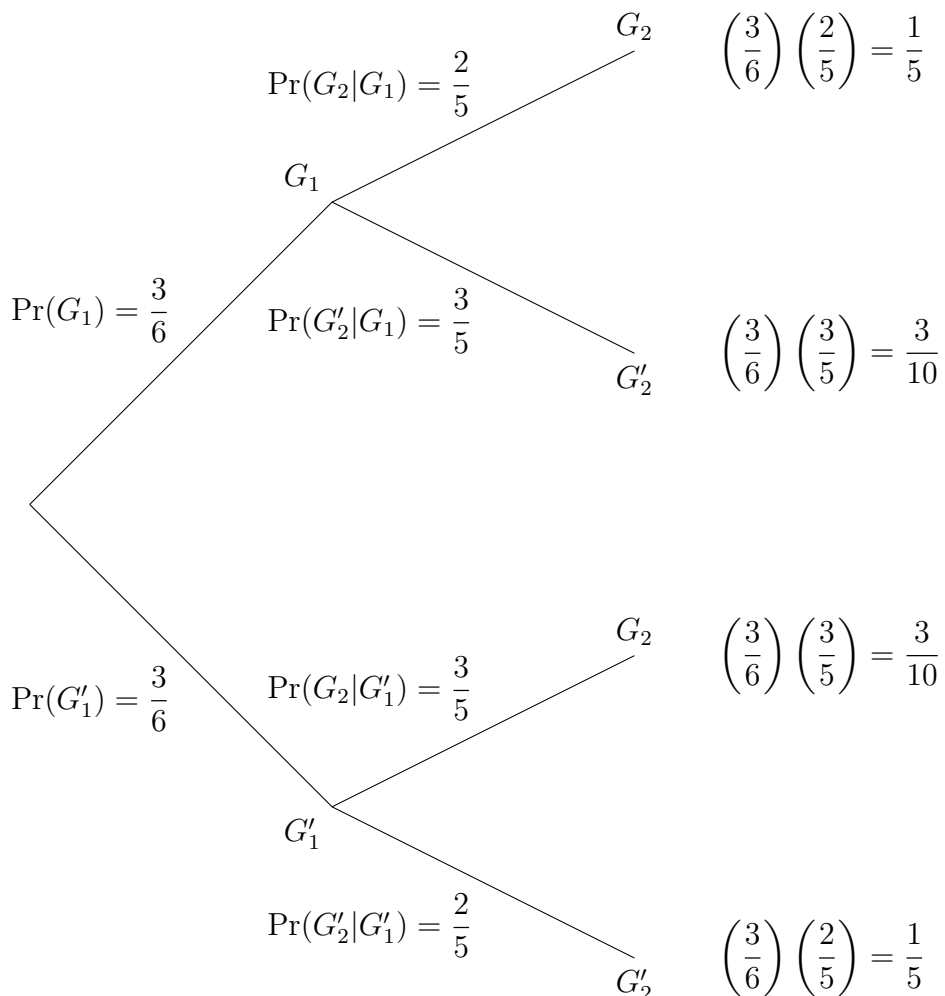


**SOLUTION TO THE QUESTION FOR MATH 343 FOR  
THE LECTURE OF 14 APRIL**

**Problem 1.** A box contains 3 green marbles and 3 purple marbles. Two are chosen at random, without replacement. What is the probability that both have the same color? What is the probability that they have different colors? Use a tree diagram.

*Solution.* Let  $G_1$  be the event that the first marble is green, and let  $G_2$  be the event that the second marble is green. Here is the diagram; justifications and conclusion afterwards.



Here are the justifications for the numbers in the diagram.

We have  $\Pr(G_1) = \frac{3}{6}$  because there are 6 marbles total, of which 3 are green. Similarly,  $\Pr(G'_1) = \frac{3}{6}$  because there are 6 marbles total, of which 3 are not green.

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*Date:* 14 April 2021.

We have  $\Pr(G_2|G_1) = \frac{2}{5}$  since, given  $G_1$ , there are now 5 marbles, of which 2 are green. Similarly,  $\Pr(G'_2|G_1) = \frac{3}{5}$  since, given  $G_1$ , there are now 5 marbles, of which 3 are not green.

We have  $\Pr(G_2|G'_1) = \frac{3}{5}$  since, given  $G'_1$ , there are now 5 marbles, of which 3 are green. Similarly,  $\Pr(G'_2|G'_1) = \frac{2}{5}$  since, given  $G'_1$ , there are now 5 marbles, of which 2 are not green.

The events on the right are disjoint, so we can add probabilities. The top and bottom ones together are the event that both marbles have the same color. Therefore the probability of this outcome is  $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$ . The probability that they have different colors is then  $1 - \frac{2}{5} = \frac{3}{5}$ . (Alternatively, it is the sum of the probabilities of the two middle branches,  $\frac{3}{10} + \frac{3}{10} = \frac{3}{5}$ .)  $\square$