Consider a RV $X$ with values in $(-\infty, \infty)$.

Assume discrete for now. For $x \in (-\infty, \infty)$, look at $P_X(x) < \text{std notation}$. (Better: $P_X(x = x)$ for $x \in \mathbb{S}$).

In both cases, called $f_X(x)$.

For example, let $X$ be the mass of a marble. $P_X(3) = \frac{5}{19}$, $P_X(4) = \frac{8}{19}$, and $P_X(x) = 0$ for all other real $x$.

Note: $\sum_{x \in \mathbb{R}} f_X(x) = 1$. Hence $f_X(3) + f_X(3.5) + f_X(4) = 1$.

For RVs with finitely many values, this is a finite sum. ($f_X(x) = 0$ for all but finitely many $x$).

The cumulative distribution function: $F_X(x) = P_X(x \leq x)$.

What is $F_X(2)$, $F_X(3.5)$, $F_X(7)$ for $X$ as above?

$P_X(X \leq 2)$, $P_X(X \leq 3.5)$, $P_X(X \leq 4)$

Fundamental Theorem: There are no marbles with mass $2$ grams or less.

$F_X(x) = \begin{cases} 0 & x < 3 \\ \frac{5}{19} & 3 \leq x < 3.5 \\ \frac{11}{19} & 3.5 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$
In general: \( \lim_{x \to -\infty} F_X(x) = 0 \) and \( \lim_{x \to \infty} F_X(x) = 1 \).

Ex. Scores on final exam in a Math 101 class were 96, 103, 111, 113, 136, 175. \( X \) is the score of a randomly chosen student.

What are \( f_X \) and \( F_X \)?

\( f_X(x) \) is nonzero only for \( x = 96, 103, 111, 113, 136, 175 \). \( f_X(111) = \frac{2}{6} \) and \( f_X(136) = \frac{1}{6} \) for \( x = 96, 103, 111, 136, 175 \).

\[
F_X(x) = \begin{cases} 
0 & x < 96 \\
\frac{1}{6} & 96 \leq x < 103 \\
\frac{2}{3} & 103 \leq x < 111 \\
\frac{3}{6} & 111 \leq x < 136 \\
1 & 136 \leq x \leq 175.
\end{cases}
\]

Ex. Suppose \( F_X(x) = \begin{cases} 
0 & x < 0 \\
\frac{1}{12} & 0 \leq x < 3 \\
1 & 3 \leq x.
\end{cases} \)

What is \( f_X \)?

\( f_X(0) = \frac{1}{12} \) and \( f_X(3) = 1 - \frac{1}{12} \).

Cumulative dist. function: \( F_X(x) = \begin{cases} 
0 & x < 0 \\
\frac{1}{12} & 0 \leq x < 1 \\
\frac{3}{4} & 1 \leq x < 2 \\
\frac{5}{12} & 2 \leq x < 3 \\
1 & 3 \leq x.
\end{cases} \)

\( P_X(x = r) = 0 \) if \( r < 0 \). \( P_X(x = 0) = \frac{1}{12} \). So \( P_X(x = 0) = \frac{1}{12} \).

Need: \( P_X(x < r) = \lim_{x \to r-} P_X(x \leq x) \).

Ex. Given table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f_X(x) )</th>
<th>( F_X(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{2}{7} )</td>
<td>( \frac{3}{7} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{4}{7} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{3}{7} )</td>
<td>1</td>
</tr>
</tbody>
</table>

Zero for all other \( x \).

Is \( f_X(x) \) legitimate? If so, what is \( F_X(x) \)?

Yes: Values of \( f_X(x) \) add up to 1, and none is \( < 0 \).
Let $X$ be a RV. Its expected value (in discrete case) or mean $\mu_X$ is the weighted average of its values.

Exam scores were 96, 103, 111, 118, 126, 175.

Mean score is $\frac{96 + \cdots + 175}{6}$.

$$E(X) = \sum_{x \in (-\infty, \infty)} x f_X(x) = \frac{1}{6} (96) + \frac{1}{6} (103) + \frac{2}{6} (111) + \frac{1}{6} (136) + \frac{1}{6} (175)$$

This really is the mean exam score.

Wednesday's class problem. $E(X)$ for $X$ result of rolling one fair die.

Recall RV $X$ with $f_X(0) = \frac{1}{2}$, $f_X(1) = 1 - \frac{1}{2}$, $f_X(x) = 0$ otherwise.

$$E(X) = 0 \cdot \frac{1}{2} + 1 \cdot \left(1 - \frac{1}{2}\right) = \frac{1}{2}$$

$$= 3 \cdot \left(1 - \frac{1}{2}\right) = \frac{3}{2}$$