

**SOLUTION TO THE QUESTION FOR MATH 343 FOR
THE LECTURE OF 28 APRIL**

Problem 1. A box contains 1 blue marble and 9 red marbles. A marble is chosen at random. Record 0 if it is red and 1 if it is blue.

- (1) Is Z a Bernoulli random variable?
- (2) Find $E(Z)$.
- (3) Find the standard deviation σ_Z .

Solution to (1). Yes: the only values are 0 and 1, each with a fixed probability. □

Solution to (2). We choose a red marble ($Z = 0$) with probability $\frac{9}{10}$ and a blue marble ($Z = 1$) with probability $\frac{1}{10}$. The formula in the book tells us that $E(Z) = \frac{1}{10}$. Here is the computation:

$$E(Z) = 0 \cdot \Pr(Z = 0) + 1 \cdot \Pr(Z = 1) = \Pr(Z = 1) = \frac{1}{10}.$$

□

Solution to (3). The formula in the book gives us the first step in the calculation

$$\sigma_Z = \sqrt{\left(\frac{1}{10}\right) \left(1 - \frac{1}{10}\right)} = \sqrt{\frac{9}{100}} = \frac{3}{10}.$$

Here is the direct computation. We have $Z^2 = Z$ since the only values of Z are 0 and 1. Therefore

$$\begin{aligned} \text{Var}(Z) &= E(Z^2) - E(Z)^2 = E(Z) - E(Z)^2 \\ &= \frac{1}{10} - \left(\frac{1}{10}\right)^2 = \frac{1}{10} - \frac{1}{100} = \frac{9}{100}, \end{aligned}$$

so

$$\sigma_Z = \sqrt{\text{Var}(Z)} = \sqrt{\frac{9}{100}} = \frac{3}{10}.$$

□