

**SOLUTION TO THE QUESTION FOR MATH 343 FOR  
THE LECTURE OF 30 APRIL**

**Problem 1.** Due to a machine being out of adjustment, widgets manufactured by Wang's Widgets Inc. have chipped paint with probability  $\frac{1}{20}$ . In a crate of 20 widgets selected at random, find the probabilities of each of the following events. Expand the binomial coefficients, for example,

$$\binom{12}{4} = \frac{12!}{4! \cdot 8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4},$$

simplify any occurrences of  $1 - \pi$  for some probability  $\pi$ , and make obvious cancellations, but don't do any other simplification or expansion.

- (1) No widgets in the crate have chipped paint.
- (2) Exactly two widgets in the crate have chipped paint.
- (3) At most two widgets in the crate have chipped paint.

The decimal approximations in the following solutions were not requested, but are given for those who calculated

*Solution to (1).*

$$\left(1 - \frac{1}{20}\right)^{20} = \left(\frac{19}{20}\right)^{20} \approx 0.35849.$$

□

*Solution to (2).*

$$\begin{aligned} \binom{20}{2} \left(\frac{1}{20}\right)^2 \left(1 - \frac{1}{20}\right)^{18} &= \frac{20!}{2! \cdot 18!} \left(\frac{1}{20}\right)^2 \left(\frac{19}{20}\right)^{18} \\ &= \left(\frac{20 \cdot 19}{1 \cdot 2}\right) \left(\frac{1}{20}\right)^2 \left(\frac{19}{20}\right)^{18} \\ &= \left(\frac{19}{2}\right) \left(\frac{1}{20}\right) \left(\frac{19}{20}\right)^{18} \approx 0.37735. \end{aligned}$$

□

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*Solution to (3).* The probability of exactly one widget having chipped paint is

$$\begin{aligned} \binom{20}{1} \left(\frac{1}{20}\right)^1 \left(1 - \frac{1}{20}\right)^{19} &= \frac{20!}{1! \cdot 19!} \left(\frac{1}{20}\right) \left(\frac{19}{20}\right)^{19} \\ &= 20 \left(\frac{1}{20}\right) \left(\frac{19}{20}\right)^{19} = \left(\frac{19}{20}\right)^{19} \approx 0.18868. \end{aligned}$$

Therefore the probability that at most two widgets in the crate have chipped paint is

$$\left(\frac{19}{20}\right)^{20} + \left(\frac{19}{20}\right)^{19} + \left(\frac{19}{20}\right) \left(\frac{1}{20}\right) \left(\frac{19}{20}\right)^{18} \approx 0.924512.$$

□