

**SOLUTION TO THE QUESTION FOR MATH 343 FOR  
THE LECTURE OF 12 MAY**

**Problem 1.** A continuous random variable  $X$  has the probability density function

$$f_X(t) = \begin{cases} 0 & t < 0 \\ e^{-t} & t \geq 0. \end{cases}$$

Find  $\Pr(X \leq 2)$  and  $\Pr(X \geq 2)$ .

*Solution.* We have

$$\begin{aligned} \Pr(X \leq 2) &= \int_{-\infty}^2 f_X(t) dt = \int_0^2 e^{-t} dt \\ &= (-e^{-t}) \Big|_0^2 = -e^{-2} - (-1) = 1 - e^{-2}. \end{aligned}$$

Since  $\Pr(X = 2) = 0$ , we then have

$$\Pr(X \geq 2) = 1 - \Pr(X \leq 2) = e^{-2}.$$

The second part can also be done directly:

$$\begin{aligned} \Pr(X \geq 2) &= \int_2^{\infty} f_X(t) dt = \int_2^{\infty} e^{-t} dt \\ &= (-e^{-t}) \Big|_2^{\infty} = \lim_{t \rightarrow \infty} e^{-t} - (-e^{-2}) = 0 + e^{-2} = e^{-2}. \end{aligned}$$

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