This file describes some mistakes commonly made in Written Homework 6.

In Problem 7.9, you are given

\[ F_X(x) = \begin{cases} 
0 & x < 0 \\
\frac{x^2}{4} & 0 \leq x < 2 \\
1 & 2 \leq x.
\end{cases} \]

When finding the 25th percentile and median, it is a waste of time to calculate \( f_X(x) = F_X'(x) \) and then solve the equations

\[ \int_{-\infty}^{p} f_X(x) \, dx = \frac{1}{4} \quad \text{and} \quad \int_{-\infty}^{m} f_X(x) \, dx = \frac{1}{2}. \]

Just solve

\[ F(p) = \frac{1}{4} \quad \text{and} \quad F(m) = \frac{1}{2}. \]

Similarly, in Problem 7.40, you are given

\[ \Pr(X < x) = F_X(x) = \begin{cases} 
1 - \frac{4}{x^2} & x > 2 \\
0 & \text{otherwise}.
\end{cases} \]

When asked for \( \Pr(X < 5) \), it is a waste of time to evaluate this as

\[ \int_{-\infty}^{5} F'_X(x) \, dx, \]

since you already have \( F_X(5) \) and \( \lim_{x \to -\infty} F_X(x) = 0 \). On an exam, this will take time away from other problems, and cost points as a result.

In Problem 7.9, given

\[ F_X(x) = \begin{cases} 
0 & x < 0 \\
\frac{x^2}{4} & 0 \leq x < 2 \\
1 & 2 \leq x,
\end{cases} \]

and being asked to solve the equation \( F(x) = \frac{1}{4} \), you must explain why \( x = -1 \) is not a solution. Otherwise, I don’t know that you correctly solved the equation \( x^2 = 1 \).

It is not permissible to use \( x \) both as a limit of integration and as the variable of integration. For example, the expressions

\[ \int_{-\infty}^{x} f_X(x) \, dx \quad \text{and} \quad \left. \left( -e^{-x/2} \right) \right|_{0}^{x} \]

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are both wrong.

There were several examples of values $F_X(x) = \Pr(X < x)$, or other probabilities, being claimed to be less than zero or greater than one. These are obviously impossible.

The correct notation for the derivative of $1 - \frac{4}{x^2}$ is

$$\frac{d}{dx} \left(1 - \frac{4}{x^2}\right).$$

The following are all wrong:

$$\frac{d}{dx} \left(\frac{4}{x^2}\right), \quad \left(1 - \frac{4}{x^2}\right) \frac{d}{dx}, \quad \text{and} \quad \left(1 - \frac{4}{x^2}\right) dx.$$

The first means

$$\frac{d}{dx} \left(1 - \frac{4}{x^2}\right) = 0 - \frac{4}{x^2} = \frac{4}{x^2}.$$ 

(Order of operations: differentiation before addition and subtraction.) The second is a differential operator, not a function. The third is a differential form, not a function.

Parentheses are required in the expression $\int_{-1}^{x} (t) \, dt$. The expression $\int_{-1}^{x} t \, dt$ is wrong. Order of operations: integration before addition and subtraction.

The expression $\frac{\gamma}{\infty}$ is meaningless. The same applies to all other algebraic operations involving $\infty$ or $-\infty$, except for $-\infty$ itself. When you mean limits, use correct notation for them:

$$\lim_{x \to \infty} \left(-\frac{8}{x}\right).$$

(The parentheses are required.)

Variables must agree. It can be

$$f_T(t) = 1 - e^{-t/2} \quad \text{or} \quad f_T(x) = 1 - e^{-x/2},$$

but never

$$f_T(x) = 1 - e^{-t/2} \quad \text{or} \quad f_T(t) = 1 - e^{-x/2}.$$ 

Chebyshev’s Inequality does not say

$$\Pr(-r \sigma_X < X - \mu_X < r \sigma_X) < \frac{1}{r^2}.$$ 

It says

$$\Pr(-r \sigma_X < X - \mu_X < r \sigma_X) > 1 - \frac{1}{r^2}.$$
When asked for a cumulative distribution function, for example,

\[ F_X(x) = \begin{cases} 
0 & x \leq -1 \\
\frac{1 - x^2}{2} & -1 < x \leq 0 \\
\frac{1 + x^2}{2} & 0 < x \leq 1 \\
1 & 1 < x,
\end{cases} \]

you must specify for which values of \( x \) each formula is valid. Formulas like \( F_X(x) = 1 - \frac{x^2}{2} \) or \( F_X(x) = \frac{1 + x^2}{2} \), without saying where each is valid, are wrong, even if both appear. Moreover, in this example, the specifications \( F_X(x) = 0 \) when \( x \leq -1 \) and \( F_X(x) = 1 \) when \( x > -1 \) are an essential part of a correct answer. They may not be left out. The same considerations apply to probability density functions.