Example 8.2 Code errors. Radio signals transmitted to earth from deep space probes are very weak against a noisy background. Suppose there is a 3% chance a transmitted bit will be misread. If 1000 bits are transmitted, what is the chance that no more than 35 of the bits were misread?

Binomial distr.; \( \pi = 0.03 \), \( n = 1000 \). \( \text{Want to know } \Pr(K \leq 35) \). Approx. with normal distr. with \( \mu = (1000)(0.03) = 30 \), \( \sigma = \sqrt{1000(0.03)(0.97)} \).

\[
 z = \frac{35 - \mu}{\sigma} = \frac{35 - 30}{\sqrt{1000(0.03)(0.97)}} \approx 0.9263.
\]

Approximate: \( \Pr(Z < z) = \Phi(z) \approx 0.8230 \).

Exact answer: \( \sum_{k=0}^{35} \binom{1000}{k} (0.03)^k (0.97)^{1000-k} \approx 0.8468 \).

"Continuity correction" gives a better approx: \( 0.8468{\text{(Misprint in book calculation.)}} \).

Example 8.3 Code errors. Let's repeat Example 8.2 on page 229 but this time with the continuity correction. We are looking for \( \Pr(K \leq 35) \) in the case where \( n = 1000 \) and \( \pi = 0.03 \).

Look at \( \Pr(Z = z) = 0 \)

\[ z = \text{z-score for } K = 35. \]

Let \( K \) be the RV giving # of bits misread.

\[ \Pr(K = 35) \approx 0.0485. \]

To get an upper bound, use the continuity correction:

\[ \Pr(Z < z) \approx 0.0485. \]

For \( \Pr(K \leq 35) \) we can use \( \Pr(Z \leq z) \approx 0.84603 \).

\[ z = \text{z-score upper for } K = 35.5 \text{ (half way between } K = 35 \text{ and } K = 36). \]

See 8.14, 8.5: read text.

General principle: If the value of some RV \( X \) is the sum of many small effects, reasonably, independently of each other but not necessarily any thing like a normal distribution, then \( X \) will be roughly normally distributed.

E.g. Large # of flips. Each individual flip has a small effect. (See above).

Repeated measurements of same thing.

Steps on 8.14, 8.8, etc. etc.
Heights of adult men, of adult women (but not of both together): sex gives a large effect.

Skip Ch. 9.

Go to Ch. 10: Several RVs at a time. One in discrete case. Main results also hold for continuous RVs, but need Math 241, 252 (probability and integration in IR).

Main concern: independence

Ex: Some of these are volumes, independent or not determined. Exp no independence

1. For a UO student: X: Math SAT score, Y: Reading test SAT score, Z: 1st year GPA.

2. In 100 rolls of a die, Y: value on 3rd roll, Z: sum of values in 3rd + 4th rolls.

3. All same time: X: temp of Eugene airport in deg C, Y: deg F. Indep?

4. On same day: X: height of Eiffel tower in m, Y: rainfall. Certain not indep.

Ex: Flip fair coin 3 times. X: # of heads, Y: # of tails. Indep.

To understand need joint distribution f(x, y).

<table>
<thead>
<tr>
<th>(X,Y)</th>
<th>f(x,y)(x,y)</th>
<th>Pr(X=x and Y=y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>1/8</td>
<td>1/8</td>
</tr>
<tr>
<td>(0,1)</td>
<td>1/8</td>
<td>1/8</td>
</tr>
<tr>
<td>(1,0)</td>
<td>1/8</td>
<td>1/8</td>
</tr>
<tr>
<td>(1,1)</td>
<td>1/8</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Pr(X+Y ≤ 2) = Pr(X=0) + Pr(Y=0) = 1/4 + 1/4 = 1/2

Pr(Y=1 | X=1) = Pr(Y=1 and X=1) / Pr(X=1) = (1/8) / (1/2) = 1/4

Problem 1. The joint probability mass function of two discrete random variables X and Y is given by the following table:

<table>
<thead>
<tr>
<th>y</th>
<th>f(x,y)(x,y)</th>
<th>X:0</th>
<th>X:1</th>
<th>X:2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3/13</td>
<td>1/13</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/13</td>
<td>2/13</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0/13</td>
<td>1/13</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Find Pr(X = 1 and Y = 2) and Pr(X + Y ≤ 4).
Expected value: \( E(g(x,y)) = \int g(x,y) f_{X,Y}(x,y) \, dx \, dy \)

(Geometric case: \( f_{X,Y}(x,y) = g(x,y) f_{X,Y}(x,y) \, dx \, dy \))

Ex. In Problem, what is \( E(X^2 - 2Y) \)?

\( \begin{align*}
\text{Get } & 11 \cdot 2 \cdot \left( \frac{1}{3^2} \right) + 1 \cdot 3 \cdot \left( \frac{2}{3^3} \right) + 1 \cdot 4 \cdot \left( \frac{1}{3^4} \right) \\
& \text{+ three more. I got } \frac{64}{13},
\end{align*} \)

\( \begin{array}{c|cccc}
& 0 & 1 & 2 & 3 \\
\hline
0 & 1/8 & 0 & 0 & 1/8 \\
1 & 0 & 1/4 & 1/4 & 0 \\
2 & 0 & 1/8 & 1/8 & 0 \\
\end{array} \)

\( E(X^2 - 2Y) = 2 \)

\( \begin{array}{c|cccc}
& 0 & 1 & 2 & 3 \\
\hline
0 & 1/5 & 0 & 0 & 1/8 \\
1 & 0 & 1/4 & (1-2)(1/4) & 1/4 \\
2 & 0 & 1/8 & (1-4)(1/4) & 0 \\
\end{array} \)

\( \begin{align*}
\text{I got:}\quad & E(X^2 - 2Y) = \frac{1}{2} \\
\text{Thm. } & 1) \text{ if } a \text{ is a constant, then } E(ax) = a \cdot E(x) \\
& W = X^2, \quad Z = -2Y \\
& E(W + Z) = E(W) + E(-Z) \\
& E(X^2 - 2Y) = E(X^2) + E(-Z) \\
& -2 E(Y). 
\end{align*} \)
Then if $X$ and $Y$ are independent, then $E(XY) = E(X)E(Y)$.

Define $X, Y$ are independent if $f_{X,Y}(x,y) = f_X(x)f_Y(y)$ for all $x$ and $y$.

Equivalently: $X, Y$ not related.

$E$ $E$, $F$ independent events. $X = 1$ if $E$; $Y = 1$ if $F$.

Then $X, Y$ are independent.