Problem 1. Your chemistry lab has bought a new balance, which measures masses with a standard deviation of 0.1 milligrams. Your lab assistant thinks the balance was miscalibrated at the factory. Four weighings of a reference object with known mass 200 grams give \( \bar{x} = 200.000095 \) grams (200 grams plus 0.095 milligrams) and a sample standard deviation \( s = 0.011 \) milligrams. Test at significance level \( \alpha = 0.05 \).

Let \( \mu \) be the random variable whose values are the results of weighing this object with the new balance.

(1) State the hypotheses.
(2) Do you use the \( z \) test or the \( t \) test (or neither)?
(3) Find the test statistic (\( z \) or \( t \)).
(4) Find the \( P \)-value. (If you use the \( t \) test and the table at the back of the book, find an interval which contains the \( P \)-value.)
(5) Do you retain or reject the null hypothesis? What does this mean in plain language?

Solution. Solution to (1): The hypotheses are:

\[ H_0 : \mu = 200. \text{ (The balance is correctly calibrated.)} \]
\[ H_a : \mu \neq 200. \text{ (The balance is miscalibrated.)} \]

Solution to (2): Use the \( z \) test. We know the population standard deviation, and the distribution of \( \bar{x} \) is approximately normal (here, in fact normal), so we can use the \( z \) test.

Solution to (3): We are assuming the null hypothesis, so we are assuming \( \mu = 200. \) Thus, in milligrams (to avoid writing too many zeroes)

\[ z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{0.095}{0.1/\sqrt{4}} = 1.9. \]

Solution to (4): Let \( Z \) be a standard normal random variable. Since this is a 2-sided test, the \( P \)-value is

\[ P(Z \leq -1.9) + P(Z \geq 1.9) = 2 P(Z \leq -1.9) \approx 2(0.02872) = 0.05744. \]

Solution to (5): The \( P \)-value is greater than \( \alpha = 0.05 \), so we don’t reject the null hypothesis. We don’t have sufficient evidence to complain to the manufacturer that balance was miscalibrated at the factory. □

Date: 4 June 2021.