This assignment is due Monday 11 May at 10:00 pm. It will probably require reading in the book ahead of the lectures.

The following problem counts as one ordinary problem.

**Problem 1** (Rudin, Chapter 10, Problem 2). Let $f$ be an entire function. Suppose that for every $a \in \mathbb{C}$, in the power series representation

$$f(z) = \sum_{n=0}^{\infty} c_{n,a}(z - a)^n,$$

there is $n \in \mathbb{Z}_{\geq 0}$ such that $c_{n,a} = 0$. Prove that $f$ is a polynomial.

**Hint:** $n! c_{n,a} = f^{(n)}(a)$.

This is Problem 2 in Chapter 10 of Rudin’s book. Rudin wrote (1) as “$f(z) = \sum_{n=0}^{\infty} c_n(z - a)^n$”. Suppressing the dependence on $a$ in the notation for the coefficients makes proper writing of both the problem and its solution awkward.

The following problem counts as one ordinary problem.

**Problem 2** (Rudin, Chapter 10, Problem 3). Suppose that $f$ and $g$ are entire functions, and that $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$. What conclusion can you draw?

The following problem counts as one ordinary problem.

**Problem 3** (Rudin, Chapter 10, Problem 4). Let $f$ be an entire function. Suppose that there are constants $A, B > 0$ and $k \in \mathbb{Z}_{>0}$ such that $|f(z)| \leq A + B|z|^k$ for all $z \in \mathbb{C}$. Prove that $f$ is a polynomial.

The following problem counts as one ordinary problem.

**Problem 4** (Rudin, Chapter 10, Problem 6). Prove that there is a region $\Omega$ such that $\exp(\Omega) = B_1(1)$. Prove that there are are many such choices of $\Omega$. Prove that, for any such choice of $\Omega$, the restriction $\exp_{\mid \Omega}$ is injective. Fix one such choice of $\Omega$, and define $\log B_1(1) \to \Omega$ to be the inverse function of $\exp_{\mid \Omega}$. Prove that $\log'(z) = z^{-1}$. Find the coefficients $a_n$ in the expansion

$$\frac{1}{z} = \sum_{n=0}^{\infty} a_n(z - 1)^n,$$

and hence find the coefficients $c_n$ in the expansion

$$\log(z) = \sum_{n=0}^{\infty} c_n(z - 1)^n.$$

In which other disks can this be done?