If \( \pi \) is irreducible, then \( C(\pi, \rho) \) is a Hilbert space, with
\[
\langle a, b \rangle \text{ defined by } b^*a \in C(\pi, \rho) \text{ so } \pi(\cdot) a \text{ weakly } \pi(\cdot) b^* \text{ converges to a scalar } \langle a, b \rangle = \lambda.
\]
Note: \( \|a\|_2 = \|a\|_2 \) for any operator between Hilbert spaces.

If \( \pi \) is irreducible, then \( C(\pi, \rho) \) is a Hilbert space.

It will help later that \( C(\pi, \rho) = 0 \) even when \( \nu \) contains \( \pi \).

\( a \in C(\pi, \rho) \). Then \( a^*a \in C(\rho) \). So \( a^*a \) is a submultiplex.

Theorem: \( a \) is a scalar multiple if \( a(\pi) \) is an isometry from the Hilbert space of \( \pi \)
to \( \pi \). So \( \pi \) is closed.

Consider the case of a continuous field \( \Phi(\pi) \) at \( \epsilon \neq 0 \) \( \pi \) in \( \pi \). Hilbert space \( H \), unitary \( \Phi(\pi) \), \( \|\Phi(\pi)\| = 1 \). [not assuming unitary]

One can then average the scalar product over the group:
\[
\langle \Phi(\pi) \rangle = \int \langle \Phi(\pi) \rangle \text{ d} \pi(\pi),
\]
where \( \pi \) is a unitary field.

We give a norm to the scalar product and \( \pi \) gives eq norm. So still true that
\( \pi \) is a direct sum of irreducible regions even in the old norm.

As a particular case, if \( \pi \) is finite and \( \pi : G \rightarrow L(\mathbb{C}) \) with \( H \) a f.d. vector space over \( \mathbb{C} \), it applies. One gets the case

Next goal: Look at reg rep \( \pi \), what is its decomposition into irreps rep \( \pi \)?

Thus is a good answer, what also leads to a description of \( C^*(G) \).

Peter: Why?
Thom: For \( \pi \) irreducible, let \( \rho \) be its fem
denominator.

Then reg rep \( \pi \) is \( \bigotimes \pi \rho \).

To begin: \( \pi : G \rightarrow L(\mathbb{C}), \pi(\cdot) \in H, \) let \( \pi(\cdot) \in C(\delta) \) be the
for \( \pi(\cdot) \in C(\delta) \) is \( \langle \pi(\cdot), \rho \rangle \). Write \( \pi(\cdot) \) as \( \eta \) in \( \pi \).

There we call matrix elements of \( \pi \). We take \( \pi \) for a
prototypical basis if \( \pi \) really is getting the matrix entries in \( \pi(\cdot) \)
with basis.

\( \mathcal{E}(\pi) \) is the linear span of these \( \pi \).

\( \mathcal{E}(\pi) \in L^0(\mathbb{C}) \) for all \( \rho \in [1, \infty] \).
Let $G$ act. Then:

1. $\pi \circ \phi \Rightarrow E_{\pi \circ \phi} = E_{\phi}$.
2. $E_{\phi}$ is right and left translation invariant.
3. $E_{\phi}$ is a $T$-ideal, where $T = L(T(G))$.
4. $\pi |_{T} : G \to L(T(G))$ and $H \subseteq T(G)$. Then $\dim(E_{\phi}) \leq \dim(H)$.

Proof:

1. Let $\pi$ be a unitary rep. with $\pi(G) = C(G) \subseteq T$. Then

   $g \mapsto u(g)$.

   Hence, $E_{\phi} = E_{\phi}$.

2. $\phi_{\pi}(h) = (\pi(h) \phi_{\pi}(g), \pi(h) \phi_{\pi}(g)) = (\phi_{\pi}(g), \phi_{\pi}(g))$.

   So, $\phi_{\pi}(g) = \phi_{\pi}(h)$.

3. Notation: $H \subseteq T$ is a unitary rep. of $G$, and $H \subseteq L(C(G))$. Notate $H$ for the value of $H$ at the corresponding rep. of $T(G)$.

   Now, $\phi_{\pi}(h) = \sum_{n} \langle \pi(h) \phi_{\pi}(g), \pi(h) \phi_{\pi}(g) \rangle$. It is a unitary matrix unit.

   Check that the dual $T_{\pi}^{\ast}$ span $E_{\pi}$:

   $\phi_{\pi}(g) = \sum_{n} \phi_{\pi}(g)$.

4. Let $e_{1}, \ldots, e_{n}$ be a minimal basis for $H$. Notate $H_{j}$ for $\langle e_{j} \rangle$ and $H_{i,j}$ for $\langle e_{i}, e_{j} \rangle$.

   Check that the dual $T_{\pi}^{\ast}$ span $E_{\pi}$:

   $\phi_{\pi}(g) = \sum_{n} \phi_{\pi}(g)$.

Proof: For $\pi$ and $G \to L(H(G))$, with $\pi(G) = C(G) \subseteq T(G)$, $E_{\pi} \subseteq E_{\phi}$, a vector subspace of $C(T(G))$.

Usually not a direct sum. We will use (implicitly) that $T(G)$ is a projector. Thus, $T(G)$ is a direct sum in right.

$P: H = \bigoplus_{j=1}^{n} H_{j}$. Then $H_{j} \subseteq C(T(G))$.

That is, $H_{j} \subseteq C(T(G))$. Thus, $H_{j} \subseteq C(T(G))$.

$\phi_{\pi}(g) = \sum_{j} \langle \pi(h) \phi_{\pi}(g), \phi_{\pi}(g) \rangle$. $\pi(G) \subseteq C(T(G))$.

To see this: $\phi_{\pi}(g) = \sum_{k} \langle \pi(h) \phi_{\pi}(g), \phi_{\pi}(g) \rangle$. $\pi(G) \subseteq C(T(G))$.

The dual forms $(\cdot, \cdot)$ are zero since $H_{j} \subseteq C(T(G))$.

$H_{j} \subseteq C(T(G))$. So,
\[ \text{Theorem 1: } \forall i \in \mathbb{N}, \exists j \in \mathbb{N} \text{ s.t. } j = 2^i. \]

\[ \text{Proof: } \text{By induction on } i. \text{ Base case: } j = 1, \text{ for } i = 0. \]

\[ \text{Inductive step: } \]

\[ \text{Suppose } n \text{ are words in } E \text{ for } k < 2^i. \]

\[ \text{For } k \geq 2^i, \text{ there exists a word in } E \text{ for } k = 2^i. \]

\[ \text{Thus, } E \text{ is a finite set.} \]

\[ \text{Q.E.D.} \]