We were proving:

Prop. $(\forall \pi) \exists g \in \mathcal{N}(\pi)$ in $\text{DVR}$ for $ZL^2(G)$.

We already saw that these facts are established.

We then need: 

Lemme (not formally stated yet but true): $\mathfrak{Z} \in L^2(G) \Rightarrow \mathfrak{Z} = \sum d_\pi \mathfrak{Z}_\pi \pi \pi^{-1}$ with convergence in $L^2(G)$.

Moreover, $d_\pi \pi^{-1} \in \text{the anti-inj} \ L^2(G) \rightarrow E_\pi$.

Given lemme, if $\mathfrak{Z} \in L^2(G)$, then $\mathfrak{Z} = \sum d_\pi \mathfrak{Z}_\pi \pi \pi^{-1} = \sum \langle \mathfrak{Z}, \pi \pi^{-1} \rangle \pi^{-1}$, using formula on each lemme for $\pi \pi^{-1}$ when $\mathfrak{Z}$ is defined.

This implies that the DVR system $(\pi \pi^{-1})_{\pi \in \mathcal{N}}$ is complete, proving prop. 24.

Proof of lemme:

It suffices to consider $\mathfrak{Z} \in E_\pi$ with $\pi \pi^{-1} \mathfrak{Z} = 0$, since $E_\pi E_\pi$ is dense in $L^2(G)$.

Case 1: $\pi = \pi$ (really $\pi \sim \pi$). $\pi \pi^{-1} \rightarrow |L|$, choose any $\pi$, $H$ as usual, define

$\pi \pi^{-1} \pi \pi^{-1}$ as usual. Then:

$$\mathfrak{Z} \pi \pi^{-1} = \int \int \int \mathfrak{Z}(h \pi)(h^{-1} \pi^{-1}) d_\pi(h) = \sum \langle \mathfrak{Z}, \pi \pi^{-1} \pi^{-1} \rangle \pi^{-1} \pi \pi^{-1} \pi^{-1}$$

Case 2: $\pi \neq \pi$. Since $\mathfrak{Z} \in E_\pi$, we want to show that $\pi \pi^{-1} \mathfrak{Z} = 0$.

We know that $d_\pi \mathfrak{Z} \pi \pi^{-1} = \mathfrak{Z}_\pi \pi \pi^{-1}$ and $\pi \pi^{-1} \mathfrak{Z} = 0$. So

$d_\pi \mathfrak{Z} \pi \pi^{-1} = d_\pi \mathfrak{Z} \pi \pi^{-1} + \pi \pi^{-1} = 0$, as desired. $

Look for completion of Cauchy sums.

Lemme $G_{\text{opt}}$: The symmetric opera $\mathfrak{Z}$ with $\mathfrak{Z} = 0$ and $\mathfrak{Z} = 0$. Notice $W_0^{-1} = W_0$. are $\gamma$-subloci for $G_{\text{opt}}$.

This property of $G_{\text{opt}}$ is called "small invariance relations", abbrev. S.I.R. Some $G_{\text{opt}}$ are not $\mathfrak{Z}$-invar.

Pf: Key: the symmetric subloci are $\gamma$-subloci. Given $V \subseteq G_{\text{opt}}$, then $V \subseteq G_{\text{opt}}$ choose $g \in G_{\text{opt}}$.

$V G \subseteq V$ which implies $V_{g^{-1}} \subseteq G_{\text{opt}}$. Hence $V G_{\text{opt}} = G_{\text{opt}}$.

Define $W_0 = \bigcap_{g \in G_{\text{opt}}} g \gamma^{-1} V_{g^{-1}}$.

Claim: $g \in G_{\text{opt}} \rightarrow g W_0 \subseteq G_{\text{opt}}$. Pf: of claims: Choose $g \in G_{\text{opt}}$. Then

$g W_0 = \bigcap_{g \in G_{\text{opt}}} g \gamma^{-1} V_{g^{-1}}$. Since $g \gamma^{-1} \subseteq V$ and $V$ is symm. This shows $g W_0 \subseteq G_{\text{opt}}$.

Claim: $g W_0 \subseteq G_{\text{opt}}$ open in $V$ under $\gamma^{-1}$ and $g W_0 \gamma^{-1}$. Take $W = W_0 \cap W_0^{-1}$. Open.
Lemma (not formally stated last time): $\mathcal{L} \in C^1(\mathbb{R}) \Rightarrow \exists \gamma = \sum d_{\gamma} (|x| \leq \xi \wedge \xi \leq \xi + \xi) \in C^1(\mathbb{R})$ satisfies $d_{\gamma}$.

Recall the fact:

- $\mathcal{L}$ is a bounded linear functional: $\mathcal{L}(x) = \sum_{n=1}^{\infty} \lambda_n x_n$, $x = (x_n)$.

Prop: The covariance matrix is positive definite.

Proof: Let $\gamma = \sum_{n=1}^{\infty} d_{\gamma} (|x| \leq \xi \wedge \xi \leq \xi + \xi) \in C^1(\mathbb{R})$.

Rmk: $\mathcal{L}$ is continuous and bounded on $C^1(\mathbb{R})$.

Corollary: For any $v_i \in C^1(\mathbb{R})$, $\mathcal{L}(v_i) = \sum_{n=1}^{\infty} \lambda_n v_i^{(n)}(0)$.

Proof: By the boundedness of $\mathcal{L}$, $\mathcal{L}(v_i)$ is bounded for each $v_i$.

Rmk: The covariance matrix is positive definite.

Corollary: For any $v_i \in C^1(\mathbb{R})$, $\mathcal{L}(v_i) = \sum_{n=1}^{\infty} \lambda_n v_i^{(n)}(0)$.

Proof: By the boundedness of $\mathcal{L}$, $\mathcal{L}(v_i)$ is bounded for each $v_i$.

Rmk: The covariance matrix is positive definite.