1. For $\text{Max}(\mathbb{Z}_\ell([G]))$ $A_\pi$, $\omega_\pi(t) = \lambda_\pi^{-1} \int \overline{f}(s) \, d\tau$, where $d\tau = \partial\pi^{-1} <t, x_\pi>$.

(Which $d\tau$ before (Wrong in Fillmore's book.) Note: $\omega_\pi(t) x_\pi = f(t) x_\pi$ (d\tau does not appear in this formula). Showing $\omega_\pi$ is a homomorphism $\omega_\pi(f \ast g) x_\pi = f \ast g \ast x_\pi = \partial\pi^{-1} \int f x_\pi \ast g x_\pi$ (using $\mathcal{N} x_\pi \ast x_\pi = \partial\pi^{-1} x_\pi$)

$\omega_\pi(f \ast g) x_\pi = \partial\pi^{-1}(f \ast g) \omega_\pi(x_\pi) = \partial\pi^{-1} f \omega_\pi(x_\pi) \ast \partial\pi^{-1} g \omega_\pi(x_\pi)$

(2) Consider $\eta_\psi \psi$ or the limit of Cesaro summation.

$c_\psi(z) = \omega_\psi(x_\pi) = c_\psi(x_\pi)$. Then $c_\psi(z) = \partial\pi^{-1} <\psi, x_\pi>$. So $c_\psi(0) = \sum \left| \frac{\partial}{\partial z} \right|^2 <\psi, \psi> < z, 0 < z < \infty$ (This is $\|\eta_\psi\|^2$.

(3) $\|\eta_\psi\|^2$.

(This is a misprint in Fillmore's book.)

Then $G$ is a $\mathbb{C}$-

\[ C^\infty(G) \approx \bigoplus \mathcal{M}_\pi (\mathbb{C}) \]

Comment: (1) $\mathcal{M}_\pi (\mathbb{C})$ gets operator norm from identification with $L^2 (\mathbb{C})$ (the point).

(2) $\psi$ is the only $\mathcal{C}^*$-norm in $\mathcal{M}_\pi (\mathbb{C})$ with given algebra structure and adjoint.

(3) If $\pi : G \to \mathcal{L}(\mathcal{H})$ is the $\mathcal{C}^*$-dual sum:

4. $\mathcal{A}$ is the set of elements $a = (a_g)_{g \in G}$ with $a_g \in \pi(A_g) V_g$ and that $\sum_{g \in G} \sum_{v \in \mathcal{V}} |a_g|^2 < \infty$. Note: $\|\sum_{g \in G} a_g \| < \infty$ for all but finitely many $a_g$.

Will need a sub-case of $C^*$-algebra

(a) $C^*(G)$ is the closure of the image of $\Lambda(G)$ under the integrated form of left regular

(b) $\| \sum_{g \in G} \pi_g (a_g) \|_\pi = \max \{ \| \pi_g (a_g) \| \}$

(c) $C^*(\pi)$ is the closure of the image of $\Lambda(G)$ under the integrated form of left regular

(d) $\| \sum_{g \in G} \pi_g (a_g) \|_\pi = \max \{ \| \pi_g (a_g) \| \}$

3. $C^*(G)$ is the closure of the image of $\Lambda(G)$ under the integrated form of left regular

4. $\| \sum_{g \in G} \pi_g (a_g) \|_\pi = \max \{ \| \pi_g (a_g) \| \}$

This is the definition of the reduced $C^*$-algebra $C^r (G)$. It is a $\mathcal{C}$-algebra that can be extended to

5. $C^*(G)$ is the closure of the image of $\Lambda(G)$ under the integrated form of left regular

6. $\| \sum_{g \in G} \pi_g (a_g) \|_\pi = \max \{ \| \pi_g (a_g) \| \}$

This is the definition of the reduced $C^*$-algebra $C^r (G)$. It is a $\mathcal{C}$-algebra that can be extended to

7. $C^*(G)$ is the closure of the image of $\Lambda(G)$ under the integrated form of left regular

8. $\| \sum_{g \in G} \pi_g (a_g) \|_\pi = \max \{ \| \pi_g (a_g) \| \}$

This is the definition of the reduced $C^*$-algebra $C^r (G)$. It is a $\mathcal{C}$-algebra that can be extended to
Proof: We will show more: (1) The subalgebra $M_{Op}$ acts on $L^2(C)$.

(2) As a conditional alg, $C^*(C)$ also $L^2(C)$ for $p < L^2(C)$ is some complete alg.

(3) $\text{dist} \in M_{Op} \in L^2(C)$ with some norm on $M_{Op}$. Usually, (2) for a few known no clean description at these norms. There is one for $p = 2$: we Hilbert-Schmidt norm on $M_{Op}$. (C) and the Hilbert space direct sum norm.

Proof of Thm. Choose a set $S$ of representatives of the classes in $C$. Use $\pi \in \mathcal{U}$ to $L_\pi$. $\pi$.

By def, we have $\pi$ where $\phi: L^2(C) \to C^*(C)$ with dense range. We know $\phi$ is continuous.

Also $L^2(C) \to L^2(C)$ is contractive with dense range, so get $\phi: L^2(C) \to C^*(C)$ contractive (though not dense range).}

The inclusions $\phi_{\pi} : C^*(C)$ are strongly contractive, bounded, and dense, so $\phi_{\pi}$ is $C^*(C)$.

Claim: $f \in L^2(C) \Rightarrow \phi(f) = \sum_{\pi \in \mathcal{U}} \pi(f) \phi_{\pi} (f)$ with $C^*$ norm convergence.

Proof of claim: We know already $f = \sum_{\pi \in \mathcal{U}} \phi_{\pi} f \pi_{\pi}$ with $L^2$ convergence.

Use $\phi$ cont and $\phi_{\pi} f \pi_{\pi}$ is a $\pi_{\pi}$-norm bounded. Claim proved.

Claim: $a \in C^*(C) \Rightarrow a = \sum_{\pi \in \mathcal{U}} \pi(a) \pi_{\pi}$ (convergence in $C^*$ norm).

Proof of claim: Let $\varepsilon > 0$. Need a finite set $F_\pi \subset S$ s.t. $\forall f \in C^*(C)$ st. $a - \phi(f) \leq \varepsilon$, choose $F_\pi \subset S$ s.t. $\forall F \pi \pi \leq \varepsilon$. Choose $F_\pi \subset S$ s.t. $\forall f \in C^*(C)$ st. $\phi(f) = \sum_{\pi \in \mathcal{U}} \phi_{\pi} f \pi_{\pi}$ with $C^*$ norm convergence.

We can compute $C^*(C) = \bigoplus_{\pi \in \mathcal{U}} \pi(C^*(C)) \pi_{\pi}$.

By compactness of $C^*$ norm, there exists some algebra norm $M_{Op} \to \pi(C^*(C)) \pi_{\pi}$.

Note: $E_\pi C^*(C) = (a\pi^2 f) \varphi^2 f(\pi_\pi)$, so $\phi(E_\pi)$ is dense in $\pi(C^*(C)) \pi_{\pi}$.

Since $E_\pi$ is self-adjoint, $\psi(E_\pi) = \pi_\pi(C^*(C)) \pi_{\pi}$.

Claim: $E_\pi$ is isomorphism from $M_{Op}$ to $\pi(C^*(C)) \pi_{\pi}$.

Since $\phi(E_\pi) = \pi_\pi$, this will imply by simplicity of $M_{Op}$ that $\pi_\pi(E_\pi)$ is injective, completing proof.
If $G$ is a locally abelian, then $\phi_G = \psi$ if $G$ is a compact group, and

$$C^*(G) \cong C^0(E)$$

($E$ with discrete topology).

This is true for all loc abelian $G$, giving $G$ its usual topology.

To extend: $C^*(G) \to C(E)$ induces an isomorphism (including homeomorphism) of the

The $G$ to be the discrete Heisenberg group: 

$$\left\{ \begin{pmatrix} 1 & m & k \\ 0 & 1 & n \end{pmatrix} : m, n, k \in \mathbb{Z} \right\}$$

with matrix norm discrete top.

Then $C^*(G)$ is the section algebra of a continuous field of $C^*$-algebras over $S^1$.

With $A_S$ being the rotation algebra, unbounded $C^*$-algeb by untraceable $U \in$ it.

$V^* = SuvV$.

The fibers are non-unital except for $S^1S^1$. (No local triviality of the bundle!)

If $S = e^{2\pi i \theta}$ and $\theta \in \mathbb{R}$, then $A_S$ is simple infinite dimensional unital

(not type I, so has bad representation theory) [an irrational rotation algebra]

If $\theta \in \mathbb{R}$ cosine, get a rational rotation algebra. It is the section algebra of

arbitrary trivial bundle over $S^1 \times S^1$ with fiber $M_n$ for suitable $n$.

But not trivial unless $S = 1$.

Elements of $C^*(G)$ can be thought of as "functions with all $G \otimes A_S$ to $S^1$.

$S^1$."