Lemma: Let \( p, q \in \mathbb{Z} \). Then \( \sum_{\omega \in \mathbb{Z}} \delta_{(\sum \omega)} = \sum_{\|\sum \omega\|^2 = \|\sum \omega\|^2} \delta_{(\sum \omega)} = 0 \quad (p, q) \neq (0, 0) \) and \( \delta_{(p, q)} = \gamma_{p, q} \) if \( p, q \neq (0, 0) \).

Let \( P \) be the set of a polynomial \( f_{m, n} \) on \( \mathbb{Z} = \{ (\sum \omega) \in \mathbb{Z}^2 : \|\sum \omega\|^2 = \|\sum \omega\|^2 \} \) and \( P_n \) be the set of polynomials of degree \( m \). The degree \( \|\sum \omega\|^2 \) is the degree of the polynomial. The following is a fundamental result:

\[
\pi \cdot \Sigma(\omega) \rightarrow L(\mathbb{R}) \quad \text{by} \quad (\pi(\omega))p(\sum \omega) = \pi(\omega) p(\sum \omega) \quad \text{(interpolation of weights \( \omega \)).}
\]

For \( \omega = \left( \begin{array}{c} e^{i\theta} \\ 0 \end{array} \right) \):

\[
\psi(\omega) = \left( \begin{array}{c} \cos(\theta) \\ \sin(\theta) \end{array} \right), \quad \xi(\omega) = \left( \begin{array}{c} -\sin(\theta) \\ \cos(\theta) \end{array} \right), \quad \zeta(\omega) = \left( \begin{array}{c} \cos(\theta) \\ \sin(\theta) \end{array} \right).
\]

Corollary: For \( \omega = \sum \omega \rightarrow \frac{1}{(\sum \omega)^{\frac{m}{2}}} \).

\( v_{(n-m)} \) are an ONB of \( P \).

For fixed \( m \), get ONB for \( P_m \).

Corollary: \( P_m \perp P_l \) for \( m \neq l \).

We want to show that \( \pi \) is well defined for \( m = 0, 1, 2, \ldots \).

Lemma: Let \( V \subset P_m \) be an invariant subspace. Then \( V \) is well defined for \( m = 0, 1, 2, \ldots \).

(These questions are the actions of the derivatives at 0 of \( Y \) and \( Z \).)

Conclude: \( \Theta \rightarrow \left( \pi(\omega) \psi(\omega) \right) \sum \omega = \pi(\omega) \psi(\omega) \left( \begin{array}{c} \cos(\theta) \sum \omega \\ \sin(\theta) \sum \omega \end{array} \right) \).

Differentiate \( \Theta : \pi(\omega) \psi(\omega) \sum \omega \):

\[
\partial_{\psi(\omega)} = \left( \begin{array}{c} \cos(\theta) \sum \omega \end{array} \right) \rightarrow \sum \omega \partial_{\sum \omega} \psi(\omega) = \sum \omega \psi(\omega) \partial_{\sum \omega} \pi(\omega).
\]

Put \( \Theta = 0 : \partial_{\omega} \left( \begin{array}{c} \cos(\theta) \sum \omega \\ \sin(\theta) \sum \omega \end{array} \right) \).

Claim: \( \Theta \) is in \( V \). Normally, need to check that the quotient at but of difference.

\( Y \) and \( Z \) are indeed, so that one gets something in \( V \). Here \( M \) is just a polynomial topology of plane convergence in the usual topology, and \( M \) is already closed.

Do some with \( Z(\Theta) \).

Conclusion: \( Z(\Theta) = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \psi(\omega) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \).

If \( \pi \) were a projection in \( Y \), then we would need to find that for \( \pi \in M \),

\[
\pi_{(n-m)}^2 \psi(\omega) \left( \begin{array}{c} \cos(\theta) \sum \omega \end{array} \right) \rightarrow \pi_{(n-m)}^2 \psi(\omega) \sum \omega.
\]

So we must prove \( \psi(\omega) \sum \omega \rightarrow \pi(\omega) \partial_{\sum \omega} \psi(\omega) \sum \omega \).

Claim: \( \pi(\omega) \partial_{\sum \omega} \psi(\omega) \sum \omega = \partial_{\sum \omega} \psi(\omega) \sum \omega \).

Corollary: \( Y \): Taking line comb shows that the quotient in the state must send \( V \) to \( M \).
**Proof:** Let $MC_{P_m}$ be a minimal inv. subalg. Choose nonzero $p_m$ to write $p_m \in \sum_{k=0}^{\infty} M_k$.

Let $k_0$ be the largest $k$ s.t. $C_k \neq 0$. Apply induction $(p_m) \subset (C_0 \oplus \cdots \oplus C_k) \subset \cdots \\
\Rightarrow \quad (C_{k+1})_m \subset \cdots$

**Note:** Can put $k_0$ from $m \to m+1$. by \(u \). Let $m$ in first $m$ with $k_0 = m$.

Same $m$, but one $m$ less. **Repeat:** After a total of $k_0$ repetitions, left with me form $\sum_{k=0}^{m} M_k$ with $\sum_{k=0}^{\infty} M_k \cong \{0\}$. 

**Conclusion:** The only $p_m \in \sum_{k=0}^{m} M_k$. 

Use the above equation, $p_m \sum_{k=0}^{m} M_k \subset \sum_{k=0}^{m} M_k$. So $\sum_{k=0}^{m} M_k \subset M$. **Repeat:** $\sum_{k=0}^{m+1} M_k \sum_{k=0}^{m} M_k$ for $k_0 = 0, 1, 2, \ldots - m$. Hence $\sum_{k=0}^{m} M_k$. So $M = P_m$.

**Theorem:** If $\sum_{k=0}^{m} M_k \not\cong \{0\}$ then $\sum_{k=0}^{m} M_k \cong \{0\}$.

For outline: Show that $\sum_{k=0}^{m} M_k \not\cong \{0\}$ in $ZC(P_m)$ is all of $ZC(P_m)$.

We have $ZC(P_m)$ is dense in $ZC(\sum_{k=0}^{m} M_k)$, this will imply they have a dense subset of $ZC(\sum_{k=0}^{m} M_k)$. If we had any other element at $(\forall \gamma)$ it would have been $P_m$

for all $m$, so no $\sum_{k=0}^{m} M_k \not\cong \{0\}$ can exist.

Recall that $\gamma, f \mapsto f(\gamma) = (\gamma, e^{i \varphi}, \eta)$ is an isomorphism.

$ZC(\sum_{k=0}^{m} M_k)$

From $ZC(\sum_{k=0}^{m} M_k)$ to $C[10, t]$.

Define $\gamma_m(\gamma) = T_m(\gamma, (\gamma, 1, 0, \ldots, 0))$. [Filling]

Need to calculate these. Take the matrix of $\gamma_m(\gamma, (\gamma, 1, 0, \ldots, 0))$ in the basis $\sum_{k=0}^{m} M_k$, $\cdots$ [Filling]

So $\gamma_m(\gamma, (\gamma, 1, 0, \ldots, 0)) = e^{i \varphi}$.

$\mathrm{sp} (\gamma_m(\gamma, (\gamma, 1, 0, \ldots, 0)))$ is the set of all $f_m(\gamma) = \sum_{k=0}^{m} x_k e^{i \varphi} .

$\text{with } a \in \mathbb{Z}_2$ and $(a, \ldots, a)$. 

(b) $\text{monic}$: $\lambda_0 = 1, \lambda_1 = \lambda, \ldots \text{ (but one depends on which } m, \text{ even or odd.)}$

Rewrite as the span of the form $\sum_{k=0}^{m} x_k e^{i \varphi}$. The span is an algebra closed under complex numbers plus $10$.

and contain the constants. (Using this Stone-Weierstrass implies result.)

Pl at claim: $\text{Constants: } f_0 = 1$. \[ Ca \text{ conl: } $f_0$, $f_1$ are real. \]

Close under mult: that $f_0 f_0 = f_2 0 f_0 2 e^{i \varphi}$.

Separate pts. $f_1(\gamma) = 2 \sin(\theta)$ separate points.

\[ \text{(Filling complete } \gamma_m(\gamma, (\gamma, 1, 0, \ldots, 0))) \]

**Arvoun:** An Invitation to $C^\ast$-Algebra

[Arvoun told me it was supposed to be "An Invitation to $C^\ast$-Algebras"]