MATH 616 (FALL 2006, PHILLIPS): OVERUSE OF ABSOLUTE VALUE SIGNS

An appropriate subtitle might be, "Why I don't like using $|S|$ for the cardinality of a set" (or for some other concept).

Absolute value signs are overused in mathematical notation. I don't object to using the same notation for two operations, functions, or whatever in two different areas where the meanings can't reasonably be confused and which are far enough apart that the notations will not collide. However, there are just too many common uses of absolute value signs, and some of them collide, sometimes badly. (See (4) below for particularly bad examples.) Moreover, there are other reasonable notations available.

(1) We write $|x|$ for the absolute value of a real or complex number x. As far as I know, this is the original use of the symbol.

(2) As a logical extension of (1), if $f \colon X \to \mathbb{C}$ is a function, one often writes $|f|$ for the function $x \mapsto |f(x)|$.

(3) A number of people write $|v|$ for the Euclidean norm $\|v\|_2$ of an element $v \in \mathbb{R}^n$ or $v \in \mathbb{C}^n$. Sometimes I have seen this notation used for other norms as well, which runs the risk of confusion with (2) and (4). (But is is still better than the physicists' practice of writing $\mathbf{v}$ for the element of $\mathbb{R}^n$ and v for its norm.)

(4) Many textbooks at the second year calculus or linear algebra level or below write $|A|$ for the determinant of a matrix A. This use is particularly bad. First, it creates confusion with (3): the norm of a matrix makes perfectly good sense (indeed, there are several of them), and so it can easily be misinterpreted. Second, this notation makes it impossible to write the absolute value of the determinant, an important combination which appears in the change of variable formula for multivariable integration. (Some people write $\|A\|$ for this, but that already has a different standard meaning, the norm.) Third, there is a perfectly good alternative, namely $\det(A)$, which has none of these problems.

(5) I have seen several books in which $|E|$ is routinely used for the Lebesgue measure of a subset $E \subset \mathbb{R}$ or $E \subset \mathbb{R}^d$. This notation is used to avoid having to give a name to Lebesgue measure (such as m, which would make m unusable for anything else). However, it collides with (6) below. As far as I am concerned, the cure is worse than the disease.

(6) Many people write $|E|$ for the cardinality of a set E. This collides with (5) above and contributes to the general overuse of the symbol. There is a perfectly good substitute with neither of these problems, namely $\mathrm{card}(E)$.

(7) Some people write $|x|$ for the degree of a homogeneous element x of a graded ring, graded module, or other graded object. I have seen this especially often with homology and cohomology theories. Again, there is a perfectly good alternative, namely $\deg(x)$.

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(8) A simplicial complex K is formally defined to be a finite set (the set of vertices) together with a collection of subsets satisfying suitable axioms. (A subset S is in the complex exactly when there is a simplex in the associated topological space whose vertices are exactly the points of S.) The associated topological space is called the *geometric realization*, and is often denoted by $|K|$.