WINTER SCHOOL COURSE: THE RADIUS OF COMPARISON OF C*-ALGEBRA CROSSED PRODUCTS BY MINIMAL HOMEOMORPHISMS

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This course is an introduction to, and background for, the "generalized Phillips-Toms conjecture", which (conjecturally) relates the mean dimension of an essentially free minimal action of a countable discrete group G (usually $\mathbb{Z}$ in the lectures) on a compact metric space X to the radius of comparison of the corresponding crossed product C*-algebra $C^*(G, X)$. Mean dimension is an invariant of dynamical systems invented with no thought of C*-algebras; its original application was to proofs of nonembeddability of one dynamical system in another. The radius of comparison is an invariant of C*-algebras invented for the purpose of analyzing counterexamples to the Elliott Classification Program in the absence of $\mathcal{Z}$-stability. The algebras on which it was originally used were not given as crossed products. The conjecture thus relates ideas from two different fields which developed entirely independently. Its formal statement is that $\mathrm{rc}(C^*(G, X)) = \frac{1}{2}\mathrm{mdim}(G, X)$.

The lectures will assume a basic course on functional analysis, including Banach algebras, spectrum, holomorphic functional calculus, and Hilbert space operator theory. They will start with the definition of a C*-algebra. We will get to the statement of the Phillips-Toms conjecture (the case $G = \mathbb{Z}$) and the definitions of the concepts appearing in it, and we hope to say a little about some of the known partial results. There will be six parts, about one hour each, with tentative titles:

(1) C*-algebras, examples, and continuous functional calculus.
(2) A brief introduction to crossed products by discrete amenable groups, mainly finite groups and $\mathbb{Z}$.
(3) Dimension of a compact space and mean dimension of a homeomorphism.
(4) Projections in C*-algebras, vector bundles, and how they are related.
(5) Cuntz comparison in C*-algebras.
(6) The generalized Phillips-Toms conjecture and some recent results.

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